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Research Article

Existence of Positive Periodic Solutions for a Class of Higher-Dimension Functional Differential Equations with Impulses

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By employing the Krasnoselskii fixed point theorem, we establish some criteria for the existence of positive periodic solutions of a class of n-dimension periodic functional differential equations with impulses, which improve the results of the literature.

1. Introduction

Some evolution processes are distinguished by the circumstance that the evolutions change very rapidly at certain instants. In mathematical simulations, impulsive delay differential equations may express several simulation processes in real world which depend on their prehistory and are subject to short time disturbances. Such processes occur in the theory of optional control, population dynamics, biotechnologies, economics, and so forth. In recent years, the existence theory of positive periodic solutions of delay differential equations with impulsive effects or without impulsive effects has been an object of active research; we refer the reader to [1–4]. For other related works on studying for impulsive delay differential equations, we refer the reader to [5–7].

In [8], Zeng et al. studied the following functional differential equations without impulses:

$$\dot{x}(t) = A(t, x(t))\, x(t) + \lambda f(t, x_t), \tag{1}$$

and obtained sufficient conditions for the existence of positive periodic solutions of (1).

Zhang et al. [9] investigated the following form:

$$\begin{gathered}\dot{x}(t) = A(t)\, x(t) + f(t, x_t), \quad t \neq \tau_k,\ k \in Z_+, \\ \Delta x|_{t=\tau_k} = I_k(x(\tau_k)).\end{gathered} \tag{2}$$

In this paper, we will consider the n-dimension differential equation with impulses as follows:

$$\begin{gathered}\dot{x}(t) = A(t, x(t))\, x(t) + \lambda f(t, x_t), \quad t \neq \tau_k,\ k \in Z_+, \\ \Delta x|_{t=\tau_k} = I_k(x(\tau_k)),\end{gathered} \tag{3}$$

where $\lambda > 0$ is a parameter, $A(t, x(t)) = \operatorname{diag}[a_1(t, x(t)), a_2(t, x(t)), \dots, a_n(t, x(t))]$, $a_i \in C(R \times R, R)$ is ω-periodic, and $f(t, x_t)$ is an operator defined on $R \times BC(R, R^n)$ (here $BC(R, R^n)$ denotes the Banach space of bounded continuous operator $\phi : R \to R^n$ with the norm $\|\phi\| = \sum_{i=1} \sup_{\theta \in R} |\phi_i(\theta)|$, where $\phi = (\phi_1, \phi_2, \dots, \phi_n)^T$). For $x \in BC$ and $t \in R$, $x_t \in BC$ is defined by $x_t(\theta) = x(t + \theta)$ for $\theta \in R$ (see [10], Zheng). Consider that $f(t + \omega, x_t) = f(t, x_t)$ and $\Delta x|_{t=\tau_k} = x(\tau_k^+) - x(\tau_k)$ (here $x(\tau_k^+)$ represents the right limit of x at the point τ_k), $I_k = (I_k^1, I_k^2, \dots, I_k^n) \in C(R_+^n, R_-^n)$, that is, x changes decreasingly suddenly at τ_k, $\omega > 0$ is a constant, R_+ and R_- are the sets of all nonnegative and nonpositive real numbers, respectively. We assume that there exists an integer $p > 0$ such that $\tau_{k+p} = \tau_k + \omega, I_{k+p} = I_k$, where $0 < \tau_1 < \tau_2 < \cdots < \tau_p < \omega$.

2. Some Preliminaries

$PC(J, R^n) = \{\phi : J \to R^n, \phi$ is continuous everywhere except at a finite number of points τ_k at which $\phi(\tau_k^+)$ and $\phi(\tau_k^-)$ exist and $\phi(\tau_k^-) = \phi(\tau_k)\}$, $J \subset R$. For each $x = (x_1, x_2, \ldots, x_n)^T \in R^n$, the norm of x is defined as $|x| = \sum_{i=1} |x_i|$.

Throughout the paper, we make the following assumptions:

(H_1) $f(t, \varphi_t) \le 0$ for all $(t, \varphi) \in R \times BC(R, R_+^n)$;

(H_2) $f_i(t, \varphi_t)$ is a continuous function of t for each $\varphi \in BC(R, R_+^n)$, $i = 1, 2, \ldots, n$;

(H_3) for any $L > 0$ and $\epsilon > 0$, there exists $\delta > 0$ such that for $\phi, \psi \in BC(R, R_+^n)$, $\|\phi\| \le L$, $\|\psi\| \le L$, and $\|\phi - \psi\| \le \delta$ imply that

$$\left|f_i(t, \phi_t) - f_i(t, \psi_t)\right| < \epsilon, \quad \forall t \in [0, \omega], \ i = 1, 2, \ldots, n. \quad (4)$$

To conclude this section, we summarize in the following a few concepts and results that will be needed in our arguments.

Definition 1. Let X be a Banach space, and let P be a closed, nonempty subset of X; P is a cone if

(i) $\alpha x + \beta x \in P$ for all $x, y \in P$ and all $\alpha, \beta \ge 0$;

(ii) $x, -x \in P$ imply $x = 0$.

Let $X = \{x = (x_1(t), x_2(t), \ldots, x_n(t))^T \in PC(R, R^n) \mid x(t+\omega) = x(t)\}$ with the norm $\|x\| = \sum_{i=1} |x_i|_0$, where $|x_i|_0 = \sup_{t\in[0,\omega]} |x_i(t)|$; then X is a Banach space.

If $x(t) = (x_1(t), x_2(t), \ldots, x_n(t))^T \in X$ is a solution of (3), then

$$\begin{aligned} x_i(t) = {} & \lambda \int_t^{t+\omega} G_i(t, u) f_i(u, x_u)\, du \\ & + \sum_{j=1}^{j=p} G_i\left(t, \tau_{m_j} + n\omega\right) I_j^i\left(x\left(\tau_{m_j}\right)\right), \end{aligned} \quad (5)$$

where

$$G_i(t, u) = \frac{\exp\left(-\int_t^u a_i(s, x(s))\, ds\right)}{\exp\left(-\int_0^\omega a_i(s, x(s))\, ds\right) - 1}, \quad i = 1, 2, \ldots, n. \quad (6)$$

See [9], Zhang et al.

It is clear that $G_i(t+\omega, u+\omega) = G_i(t, u)$, for all $(t, u) \in R^2$, and by (H_1),

$$G_i(t, u) f_i(u, \varphi_u) \ge 0 \quad (7)$$

for $(t, u) \in R^2$ and $(u, \varphi) \in R \times BC(R, R_+^n)$.

Define for $i = 1, 2, \ldots, n$,

$$\begin{aligned} A_i &:= \min_{0\le t\le u\le\omega} \left|G_i(t, u)\right| = \frac{\exp\left(-\int_0^\omega a_i(s, x(s))\, ds\right)}{1 - \exp\left(-\int_0^\omega a_i(s, x(s))\, ds\right)}, \\ B_i &:= \max_{0\le t\le u\le\omega} \left|G_i(t, u)\right| = \frac{1}{1 - \exp\left(-\int_0^\omega a_i(s, x(s))\, ds\right)}, \\ A &:= \min_{1\le i\le n} A_i, \qquad B := \max_{1\le i\le n} B_i. \end{aligned} \quad (8)$$

Let

$$\begin{aligned} K = \Big\{ & x(t) = \left(x_1(t), x_2(t), \ldots, x_n(t)\right)^T \in X : x_i(t) \ge \sigma\left|x_i\right|_0, \\ & i = 1, 2, \ldots, n\Big\}, \end{aligned} \quad (9)$$

where $\sigma = A/B \in (0, 1)$. It is not difficult to verify that K is a cone in X. We define an operator $\Phi : X \to X$ as follows:

$$(\Phi x)(t) = \left((\Phi_1 x)(t), (\Phi_2 x)(t), \ldots, (\Phi_n x)(t)\right)^T, \quad (10)$$

where

$$\begin{aligned} (\Phi_i x)(t) = {} & \lambda \int_t^{t+\omega} G_i(t, u) f_i(u, x_u)\, du \\ & + \sum_{j=1}^{j=p} G_i\left(t, \tau_{m_j} + n\omega\right) I_j^i\left(x\left(\tau_{m_j}\right)\right). \end{aligned} \quad (11)$$

Then, it can be immediately obtained from the assumptions (H_2) and (H_3) that the operator Φ is completely continuous. On the other hand, it is not difficult to check that $x^*(t) = (x_1^*(t), x_2^*(t), \ldots, x_n^*(t))^T$ is a positive ω-periodic solution of (3) if and only if $x^*(t)$ is a fixed point of the operator Φ.

Before stating the main results, we shall give some important lemmas.

Lemma 2. *The mapping Φ maps K into K, that is, $\Phi K \subset K$.*

Proof. For any $x \in K$, it is easy to see that $\Phi x \in X$. From (11), we have

$$\begin{aligned} \left|(\Phi_i x)\right|_0 \le {} & \lambda B_i \int_0^\omega \left|f_i(u, x_u)\right| du \\ & + B_i \sum_{j=1}^{j=p} \left|I_j^i\left(x\left(\tau_{m_j}\right)\right)\right|. \end{aligned} \quad (12)$$

Noting that $G_i(t, u) f_i(u, x_u) \ge 0$, we can also obtain

$$\begin{aligned} (\Phi_i x)(t) & \ge \lambda A_i \int_0^\omega \left|f_i(u, x_u)\right| du + A_i \sum_{j=1}^{j=p} \left|I_j^i\left(x\left(\tau_{m_j}\right)\right)\right| \\ & \ge \frac{A_i}{B_i} \left|(\Phi_i x)\right|_0 \\ & \ge \sigma \left|(\Phi_i x)\right|_0. \end{aligned} \quad (13)$$

Hence, $\Phi K \subset K$. The proof is complete. □

Lemma 3. *Let X be a Banach space, and let K be a cone in X. Suppose that Ω_1 and Ω_2 are open subsets of X such that $0 \in \Omega_1 \subset \overline{\Omega_1} \subset \Omega_2$. Suppose that*

$$\Phi : K \cap \left(\overline{\Omega_2} \setminus \Omega_1\right) \longrightarrow K \quad (14)$$

is a completely continuous operator and satisfies either

(1)

$$\begin{aligned} &\|\Phi u\| \leq \|u\|, \quad \forall u \in K \cap \partial\Omega_1; \\ &\|\Phi u\| \geq \|u\|, \quad \forall u \in K \cap \partial\Omega_2; \end{aligned} \tag{15}$$

or

(2)

$$\begin{aligned} &\|\Phi u\| \leq \|u\|, \quad \forall u \in K \cap \partial\Omega_2; \\ &\|\Phi u\| \geq \|u\|, \quad \forall u \in K \cap \partial\Omega_1. \end{aligned} \tag{16}$$

Then, Φ has a fixed point in $K \cap (\overline{\Omega_2} \setminus \Omega_1)$.

The proof of Lemma 3 can be found in [11], Guo et al.

Lemma 4. *Assume that (H_1)–(H_3) hold and there exists $\eta > 0$, such that*

$$\int_0^\omega |f(s,\phi_s)|\, ds \geq \eta \|\phi\|, \quad \text{for } \phi \in K. \tag{17}$$

Then,

$$\|\Phi x\| \geq \lambda A \eta \|x\|. \tag{18}$$

Proof. If $x \in K$, then

$$\begin{aligned} (\Phi_i x)(t) &\geq \lambda A_i \int_t^{t+\omega} |f_i(u,x_u)|\, du \\ &\quad + A_i \sum_{j=1}^{j=p} \left| I_j^i \left(x\left(\tau_{m_j}\right)\right)\right| \\ &\geq \lambda A_i \int_t^{t+\omega} |f_i(u,x_u)|\, du. \end{aligned} \tag{19}$$

Thus, we have

$$\begin{aligned} \|\Phi x\| &= \sup_{t\in R} \sum_{i=1} \left|(\Phi_i x)(t)\right| \\ &\geq \sum_{i=1} \lambda A_i \int_0^\omega |f_i(u,x_u)|\, du \\ &\geq \lambda A \int_0^\omega |f(u,x_u)|\, du \geq \lambda A \eta \|x\|. \end{aligned} \tag{20}$$

□

Lemma 5. *Assume that (H_1)–(H_3) hold and let $r > 0$, if there exists a sufficiently small $\epsilon > 0$ such*

$$\int_0^\omega |f(s,\phi_s)|\, ds \leq \epsilon r, \qquad \sum_{j=1}^{j=p} \left|I_j(\phi)\right| \leq \epsilon r, \qquad \text{for } \phi \in K \cap \partial\Omega_r. \tag{21}$$

Then,

$$\|\Phi x\| \leq (\lambda + 1) B \epsilon \|x\|, \quad \text{for } x \in K \cap \partial\Omega_r. \tag{22}$$

Proof. For any $x \in K \cap \partial\Omega_r$,

$$\begin{aligned} \|\Phi x\| &= \sup_{t\in R} \sum_{i=1} \left|(\Phi_i x)(t)\right| \\ &\leq \sum_{i=1} \lambda B_i \int_0^\omega |f_i(s,x_s)|\, ds \\ &\quad + \sum_{i=1} B_i \sum_{j=1}^{j=p} \left| I_j^i \left(x\left(\tau_{m_j}\right)\right)\right| \\ &\leq \lambda B \int_0^\omega |f(s,x_s)|\, ds \\ &\quad + B \sum_{j=1}^{j=p} \left| I_j \left(x\left(\tau_{m_j}\right)\right)\right| \\ &\leq (\lambda + 1) B\epsilon \|x\|. \end{aligned} \tag{23}$$

□

3. Main Results

For the sake of convenience, we introduce the following notations:

$$\begin{aligned} f^\alpha &= \lim_{x\in K} \sup_{\|x\|\to\alpha} \frac{\int_0^\omega |f(s,x_s)|\, ds}{\|x\|}, \\ f_\alpha &= \lim_{x\in K} \inf_{\|x\|\to\alpha} \frac{\int_0^\omega |f(s,x_s)|\, ds}{\|x\|}, \\ I^\alpha &= \lim_{x\in K} \sup_{\|x\|\to\alpha} \frac{\sum_{j=1}^{j=p} \left|I_j(x)\right|}{\|x\|}, \\ I_\alpha &= \lim_{x\in K} \inf_{\|x\|\to\alpha} \frac{\sum_{j=1}^{j=p} \left|I_j(x)\right|}{\|x\|}, \end{aligned} \tag{24}$$

where α denotes either 0 or ∞.

Theorem 6. *Assume that (H_1)–(H_3) hold and*

$$\begin{aligned} &(P_1) \quad f_\infty = \infty, \\ &(P_2) \quad f^0 = I^0 = 0; \end{aligned} \tag{25}$$

then (3) *has at least one positive ω-periodic solution.*

Proof. By (P_2), for any $\epsilon_1, \epsilon_1 > 0$ there exists $r_2 > 0$, such that

$$\begin{aligned} &\int_0^\omega |f(s,\phi_s)|\, ds \leq \epsilon_1 \|\phi\| \leq \epsilon_1 r_2, \\ &\sum_{j=1}^{j=p} \left|I_j(\phi)\right| \leq \epsilon_2 \|\phi\| \leq \epsilon_2 r_2. \end{aligned} \tag{26}$$

Choose $\epsilon = \max\{\epsilon_1, \epsilon_1\}$, satisfying $0 < \epsilon < (1/(\lambda+1)B)$, by Lemma 5, we have

$$\begin{aligned} &\|\Phi x\| \leq (\lambda + 1) B\epsilon \|x\| \leq \|x\|, \\ &\text{for } x \in K \cap \partial\Omega_2. \end{aligned} \tag{27}$$

Next, by (P_2), there exists $r_3 > r_2 > 0$, such that

$$\int_0^\omega |f(s,\phi_s)|\,ds \geq \eta\|\phi\|, \quad \text{for } \phi \in K,\ \|\phi\| \geq r_3, \tag{28}$$

where $\eta > 0$ is chosen, so that $\lambda A\eta > 1$. It follows from Lemma 4 that

$$\|\Phi x\| \geq \lambda A\eta\|x\| > \|x\|, \quad \text{for } x \in K \cap \partial\Omega_3. \tag{29}$$

It follows from Lemma 3 that (3) has a positive ω-periodic solution satisfying $r_2 \leq \|x\| \leq r_3$. □

Theorem 7. *Assume that (H_1)–(H_3) hold and*

$$(P_3)\quad f_0 = \infty,$$
$$(P_4)\quad f^\infty = I^\infty = 0; \tag{30}$$

then (3) *has at least one positive ω-periodic solution.*

Proof. Since $f_0 = \infty$, one can find an $r_0 > 0$, such that

$$\int_0^\omega |f(s,\phi_s)|\,ds \geq \eta\|\phi\|, \quad \text{for } \phi \in K,\ 0 < \|\phi\| \leq r_0, \tag{31}$$

where $\eta > 0$ is chosen so that $\lambda A\eta > 1$. It follows from Lemma 4 that

$$\|\Phi x\| \geq \lambda A\eta\|x\| > \|x\|, \quad \text{for } x \in K \cap \partial\Omega_{r_0}. \tag{32}$$

By (P_4), we know that there exists $N_1 > r_0$ and $\epsilon_1, \epsilon_2 > 0$ such that

$$\int_0^\omega |f(s,\phi_s)|\,ds \leq \epsilon_1\|\phi\|, \quad \sum_{j=1}^{j=p}|I_j(\phi)| \leq \epsilon_2\|\phi\|, \quad \text{for } \phi \in K,\ \|\phi\| \geq N_1. \tag{33}$$

Choose $\epsilon = \max\{\epsilon_1, \epsilon_1\}$, satisfying $0 < \epsilon < 1/(2(\lambda+1)B)$; then

$$\int_0^\omega |f(s,\phi_s)|\,ds \leq \epsilon\|\phi\|, \quad \sum_{j=1}^{j=p}|I_j(\phi)| \leq \epsilon\|\phi\|. \tag{34}$$

Take

$$r_1 > N_1 + 1 + 2B\sup_{\phi\in K,\|\phi\|<N_1}\left[\lambda\int_0^\omega |f(s,\phi_s)|\,ds + \sum_{j=1}^{j=p}|I_j(\phi)|\right], \tag{35}$$

$$\begin{aligned}\|\Phi x\| &\leq B\left[\lambda\int_0^\omega |f(s,\phi_s)|\,ds + \sum_{j=1}^{j=p}|I_j(\phi)|\right]\\ &= B\left[\rho(I_1) + \rho(I_2)\right]\\ &\leq \frac{r_1}{2} + \frac{\|x\|}{2} = \|x\|, \quad \text{for any } x \in K \cap \partial\Omega_{r_1},\end{aligned} \tag{36}$$

where $\rho(I_1) = [\lambda\int_0^\omega |f(s,\phi_s)|ds + \sum_{j=1}^{j=p}|I_j(\phi)|]|_{x\in I_i}$, $i = 1, 2$, and $I_1 = \{x \in K,\ \|x\| < N_1\}$, $I_2 = \{x \in K,\ \|x\| \geq N_1\}$.

This implies that $\|\Phi x\| \leq \|x\|$, for any $x \in K \cap \partial\Omega_{r_1}$.

Therefore, (3) has at least one positive ω-periodic solution. □

Theorem 8. *Suppose that*

(P_5) there exists $d_2 > 0$, such that $\int_0^\omega |f(s,\phi_s)|ds < d_1/\lambda A$, for $\sigma d_1 \leq \|\phi\| \leq d_1$,

(P_6) there exists $d_2 > 0$, such that $\int_0^\omega |f(s,\phi_s)|ds < d_2/2\lambda B$, $\sum_{j=1}^{j=p}|I_j(\phi)| \leq d_2/2B$, for $\|\phi\| < d_2$

hold; then (3) *has at least one positive ω-periodic solution.*

Proof. Without loss of generality, we may assume that $d_2 < d_1$. If $x \in K \cap \partial\Omega_{d_2}$, then by (P_6), one can get

$$\|\Phi x\| \leq \lambda B\frac{d_2}{2\lambda B} + B\frac{d_2}{2B} = d_2 = \|x\|, \tag{37}$$

in particular, $\|\Phi x\| < \|x\|$, for all $x \in K \cap \partial\Omega_{d_2}$.

On the other hand, by (P_5), one has

$$\|\Phi x\| \geq \lambda A\int_0^\omega |f(s,x_s)|\,ds > \lambda A\frac{d_1}{\lambda A} = d_1 = \|x\|, \quad \text{for } x \in K \cap \partial\Omega_{d_1}. \tag{38}$$

Therefore, (3) has at least one positive ω-periodic solution. □

Theorem 9. *If*

(P_7) $f^0 = \alpha_1 \in [0, 1/2\lambda B)$, $I_0 = \alpha_2 \in [0, 1/2B)$;

(P_8) $f_\infty = \beta_1 \in (1/\lambda A\sigma, \infty)$

hold, then (3) *has at least one positive ω-periodic solution.*

Proof. By assumption (P_7), for $\epsilon = \min\{(1/2\lambda B) - \alpha_1, 1/2B - \alpha_2\} > 0$, there exists a sufficiently small $d_2 > 0$ such that

$$\frac{\int_0^\omega |f(s, x_s)|\,ds}{\|x\|} < \alpha_1 + \epsilon < \frac{1}{2\lambda B},$$
$$\frac{\sum_{j=1}^{j=p} |I_j(x)|}{\|x\|} < \alpha_2 + \epsilon < \frac{1}{2B}, \quad \text{for } \|x\| \le d_2; \tag{39}$$

that is

$$\int_0^\omega |f(s, x_s)|\,ds < \frac{d_2}{2\lambda B},$$
$$\sum_{j=1}^{j=p} |I_j(x)| < \frac{d_2}{2B}, \quad \text{for } \|x\| \le d_2. \tag{40}$$

So, (P_6) is satisfied.

By assumption (P_8), for $\epsilon = \beta_1 - 1/\lambda A\sigma$, there exists a sufficiently large $d_1 > 0$ such that

$$\frac{\int_0^\omega |f(s, x_s)|\,ds}{\|x\|} > \beta_1 - \epsilon = \frac{1}{\lambda A\sigma}, \quad \text{for } \sigma d_1 \le \|x\| \le d_1, \tag{41}$$

that is

$$\int_0^\omega |f(s, x_s)|\,ds > \frac{1}{\lambda A\sigma}\|x\| \ \ge\ \frac{1}{\lambda A\sigma}\sigma d_1 = \frac{d_1}{\lambda A}, \tag{42}$$

therefore, (P_5) holds. By Theorem 8, we complete the proof. □

Theorem 10. *If*

(P_9) $f_0 = \alpha_3 \in (1/\lambda A\sigma, \infty)$;

(P_{10}) $f^\infty = \beta_2 \in [0, 1/2\lambda B)$, $I_\infty = \beta_3 \in [0, 1/2B)$

hold, then (3) *has at least one positive ω-periodic solution.*

Proof. By (P_9), for $\epsilon = \alpha_3 - (1/\lambda A\sigma) > 0$, there exists a sufficiently small $d_1 > 0$, such that

$$\frac{\int_0^\omega |f(s, x_s)|\,ds}{\|x\|} > \alpha_3 - \epsilon = \frac{1}{\lambda A\sigma}, \quad \text{for } 0 < \|x\| \le d_1, \tag{43}$$

that is

$$\int_0^\omega |f(s, x_s)|\,ds > \frac{\sigma d_1}{\lambda A} = \frac{d_1}{\lambda A}, \quad \text{for } \sigma d_1 \le \|x\| \le d_1. \tag{44}$$

Again, By (P_{10}), for $\epsilon = \min\{(1/2\lambda B) - \beta_2, (1/2B) - \beta_3\} > 0$, there exists a sufficiently small $d > 0$ such that

$$\frac{\int_0^\omega |f(s, x_s)|\,ds}{\|x\|} < \beta_2 + \epsilon, \qquad \frac{\sum_{j=1}^{j=p} |I_j(x)|}{\|x\|} < \beta_3 + \epsilon, \tag{45}$$

that is

$$\frac{\int_0^\omega |f(s, x_s)|\,ds}{\|x\|} < \frac{1}{2\lambda B}, \qquad \frac{\sum_{j=1}^{j=p} |I_j(x)|}{\|x\|} < \frac{1}{2B}, \quad \text{for } \|x\| > d. \tag{46}$$

In the following, we consider two cases to prove (P_6) to be satisfied:

(i) $\int_0^\omega |f(s, x_s)|ds + \sum_{j=1}^{j=p} |I_j(x)| < \infty$;

(ii) $\int_0^\omega |f(s, x_s)|ds = \infty$, $\sum_{j=1}^{j=p} |I_j(x)| = \infty$.

The bounded case is clear. If case (ii) is valid, then there exists $y \in BC(R, R_+^n)$, $\|y\| = d_2 > d$ such that

$$\int_0^\omega |f(s, x_s)|\,ds \le \int_0^\omega |f(s, y_s)|\,ds,$$
$$\sum_{j=1}^{j=p} |I_j(x)| < \sum_{j=1}^{j=p} |I_j(y)|, \tag{47}$$
$$\text{for } 0 < \|x\| \le \|y\| = d_2.$$

Since $\|y\| = d_2 > d$, then we have

$$\int_0^\omega |f(s, x_s)|\,ds < \frac{\|y\|}{2\lambda B} = \frac{d_2}{2\lambda B},$$
$$\sum_{j=1}^{j=p} |I_j(x)| < \frac{\|y\|}{2B} = \frac{d_2}{2B}, \tag{48}$$
$$\text{for } 0 < \|x\| \le d_2,$$

which implies that condition (P_6) holds. By Theorem 8, we complete the proof. □

Corollary 11. *If one of the following pairs*

(P_1) and (P_2); (P_3) and (P_4); (P_5) and (P_6); (P_7) and (P_8); (P_9) and (P_{10});

(P_1) and (P_7); (P_2) and (P_8); (P_3) and (P_{10}); (P_4) and (P_9)

is valid, then system (3) *has at least one positive ω-periodic solution.*

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