RELATIVE PRECISION OF MINIMUM CHI-SQUARE AND MAXIMUM LIKELIHOOD ESTIMATES OF REGRESSION COEFFICIENTS

JOSEPH BERKSON
MAYO CLINIC

Professor Doob [5], in the charming introduction to his paper delivered at the last Berkeley Symposium, remarked that physicists write like physicists, while mathematicians write like mathematicians and only for posterity. I have been given to wonder at times, for whom it is that mathematical statisticians write, and specifically whether it is for statisticians. The mathematician, we know from a witty and authoritative essay by Bertrand Russell [10], never knows what he is talking about—that is, he deals with generalities, not specificities—and he never knows whether what he is saying is true—that is, he is concerned with logical consistency, not physical facts. This ineffable disinterestedness in the tawdry realities of the physical world, mathematics shares with the sonata, abstract nonrepresentational sculpture, and the classic ballet. It does not share it with statistics. Statistics, however you define it, is very much earthbound and deals with real observable data; what is statistically true must be literally verifiably true for such data.

These remarks are prompted by an experience, which I shall presently describe, with a problem that comes to the laboratory quite frequently, and for which I presented a particular solution [1]. Specifically, in respect to any relevant consideration, so far as I could find, this solution was easier and more satisfactory than the standard one advanced by most mathematical statisticians, but it met a stone wall of opposition in the form of general theorems quoted by these mathematicians. Specifically, it appeared, the method might be fine, but generally speaking, it was no good at all. These theorems, I found when I examined the matter, stated that certain things had been proved for "large samples." Further examination disclosed, however, that if these theorems were valid for large samples, they must refer to *infinitely* large samples, which is to say, samples so large that no statistician ever gets them, at least not on this unpleasant earth.

The problem I refer to is that of bioassay, in which the potency of a drug is estimated from an experiment in which a number of animals are exposed to a series of increasing concentrations of the drug, the fraction of animals succumbing at each dose is noted, and to these fractions a regression function is fitted. The fraction affected, plotted against the dosage, the latter measured logarithmically, in certain instances follows a sigmoidal shaped curve and, as is well known, a widely used method is to fit the integrated normal curve to the data by maximum likelihood,