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IS THE LIGHT TOO LIGHT?*

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Abstract. The gravitational interaction of light is analyzed considering its dual characteristic nature, i.e., as an (electromagnetic) wave or as a particle (photon). Considered as an electromagnetic wave, the light can be source of gravitational waves belonging to the larger class of exact solutions of Einstein field equations which are invariant for a non-Abelian two-dimensional Lie algebra of Killing fields. It is shown that in the would be quantum theory of gravity they correspond to spin-1 massless particles.

1. Introduction

As described in **Quantum Electrodynamics** (QED), photon-photon scattering can occur through the creation and annihilation of virtual electron-positron pairs and may even lead to collective photon phenomena. Photons also interact gravitationally but the gravitational scattering of light by light has been much less studied. Purely general relativistic treatments of electromagnetic wave interactions have been made resulting in exact solutions [12, 13], but these calculations are different from pure scattering processes and do not address the interaction at single photon level. It is not clear to what extent, calculations of the gravitational cross-section using **Quantum Filed Theory** (QFT) methods are consistent with classical **General Relativity** (GR). First studies go back to Tolman, Ehrenfest and Podolsky

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[25] and, later, to Wheeler [27] who analysed the gravitational field of light beams and the corresponding geodesics in the linear approximation of Einstein equations. They discovered that null rays behave differently according whether they propagate parallel or antiparallel to a steady, long, straight beam of light, but they did not provide a physical explanation of this fact. Later, Barker, Bathia and Gupta [2], following a previous analysis of Barker, Gupta and Haracz [4], analyzed in QED the photon-photon interaction through the creation and annihilation of a virtual graviton in the center-mass system and they found that the interaction have eight times the "Newtonian" value plus a polarization dependent repulsive contact interaction and also obtained the gravitational cross sections for various photon polarization states. Results of Tolman, Erhenfest, Podolsky (TEP) and Wheeler were clarified in part by Faraoni and Dumse [11], in the setting of classical pure General Relativity, using an approach based on a generalization to null rays of the gravitoelectromagnetic Lorentz force of linearized gravity. They also extended the analysis to the realm of exact PP-wave solutions of the Einstein equations. After Barker, Bathia and Gupta, photon-photon scattering due to self-induced gravitational perturbations on a Minkowski background has been analyzed by Brodin, Eriksson and Marklund [2,4] solving the Einstein-Maxwell system perturbatively to third order in the field amplitudes and confirming the dependence of differential gravitational cross section on the photon polarizations.

Since the problem of the gravitational interaction of two photons is still unsolved, it appears necessary to take into full account the nonlinearity of Einstein's equations, just as in the case of gravitational waves generated by strong sources [9,24]. This is the case, for example, when the source is a coalescing binary from which a secondary wave (called the Christhodoulu memory) is generated via the non linearity of Einstein's field equations. The memory seems to be too weak to be detected from the present generation of interferometers (even if the frequency ϖ is in the optimal band for LIGO/VIRGO[1] interferometers). However, the Christodoulou memory is of the same order as the linear effects related to the same source, thus stressing the relevance of the nonlinearity of the Einstein's equations also from an experimental (LIGO/VIRGO) point of view.

2. Linearized Einstein Theory

A gravitational field $g = g_{\mu\nu}(x)\,\mathrm{d}x^\mu \mathrm{d}x^\nu$ is said to be **locally weak** if there exists a (harmonic) coordinates system and a region $M' \subset M$ of space-time M in which the following conditions hold

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \qquad |h_{\mu\nu}| << 1, \qquad |h_{\mu\nu,\alpha}| << 1. \tag{1}$$

[1]LIGO is the Laser Interferometer Gravitational Wave Observatory (the name of USA Laser Inerferometer) and VIRGO is the name of Italian Laser Interferometer

As it is known, in the weak field approximations in a harmonic coordinates system the **Einstein field equations** read

$$\Box h_{\mu\nu} = 0. \tag{2}$$

The choice of the harmonic gauge plays a key role in deriving equation (2) and no other special assumption either on the form or on the analytic properties of the perturbation h has been done. For globally **square integrable solutions** of the wave-equation (2) (that is, solutions which are square integrable everywhere on M), with a suitable gauge transformation preserving the harmonicity of the coordinate system and the "weak character" of the field, one can always kill the "**spin-0**" and "**spin−1**" components of the gravitational waves. However, in the following we will meet some interesting solutions which do not belong to this class.

2.1. Gravitoelectromagnetism

A slightly different point of view, which is useful in clarifying the nature of spin of gravitational waves is provided by the **gravitoelectromagnetism** (GEM), (see, e.g., [14]). In this scheme one tries to exploit as much as possible the similarities between the Maxwell and the linearized Einstein equations. To make this analogy evident it is enough to write a weak gravitational field fulfilling conditions (1) in the GEM form (see, e.g. [14, 18])

$$\mathrm{d}s^2 = c^2(1 + 2\frac{\Phi^{(g)}}{c^2})\mathrm{d}t^2 + \frac{4}{c}(\mathbf{A}_{(g)} \cdot \mathrm{d}\mathbf{x})\mathrm{d}t - (1 - 2\frac{\Phi^{(g)}}{c^2})\delta_{ij}\mathrm{d}x^i\mathrm{d}x^j \tag{3}$$

with

$$h_{00} = \frac{4\Phi^{(g)}}{c^2}, \qquad h_{0i} = -\frac{4A_i^{(g)}}{c^2}$$

(in this section the speed of light c will be explicitly written). Hereafter the spatial part of four-vectors will be denoted in bold and the standard symbols of three-dimensional vector calculus will be adopted. In terms of $\Phi_{(g)}$ and $\mathbf{A}^{(g)}$ the harmonic gauge condition reads

$$\frac{1}{c}\frac{\partial \Phi^{(g)}}{\partial t} + \frac{1}{2}\nabla \cdot \mathbf{A}^{(g)} = 0 \tag{4}$$

and, once the gravitoelectric and gravitomagnetic fields are defined in terms of GEM potentials, as

$$\mathbf{E}^{(\mathbf{g})} = -\nabla\Phi^{(g)} - \frac{1}{2c}\frac{\partial \mathbf{A}^{(g)}}{\partial t}, \qquad \mathbf{B}^{(\mathbf{g})} = \nabla \wedge \mathbf{A}^{(g)} \tag{5}$$

one finds that the linearized Einstein equations resemble the Maxwell equations. Consequently, being the dynamics fully encoded in Maxwell-like equations, the GEM formalism describes the physical effects of the vector part of the gravitational field. The situations which are usually described in this formalism are, typically, static: in fact, when this assumption is dropped, GEM gravitational waves are also possible.

Then, the gravitoelectric and the gravitomagnetic components of the metric are given by

$$E^{(g)}_{\mu} = F^{(g)}_{\mu 0}, \qquad B^{(g)\mu} = -\varepsilon^{\mu 0 \alpha\beta} F^{(g)}_{\alpha\beta}/2$$

where

$$F^{(g)}_{\mu\nu} = \partial_\mu A^{(g)}_\nu - \partial_\nu A^{(g)}_\mu, \qquad A^{(g)}_\mu = -h_{0\mu}/4 = (-\Phi^{(g)}, \mathbf{A}^{(g)})$$

- The first order geodesic motion for a **massive particle** in the light beam gravitational field is determined by the force

$$\mathbf{f}^{(g)} = -2\mathbf{E}^{(g)} - 4\mathbf{u} \wedge \mathbf{B}^{(g)}$$

  where $\mathbf{u}$ is the velocity of the particle.
- The first order geodesic motion for a *photon* propagating, in the light beam gravitational field, parallel(anti) to z-axis ($u_j = \pm\delta_{j3}$) is slightly different

$$\mathbf{f}^{(g)} = -4\left(\mathbf{E}^{(g)} + \mathbf{u} \wedge \mathbf{B}^{(g)}\right).$$

3. Strong Gravitational Fields

In previous papers [5–7, 19–21]) a family of exact solutions g of Einstein field equations, representing the gravitational wave generated by a beam of light, has been explicitly written

$$g = 2f(\mathrm{d}x^2 + \mathrm{d}y^2) + \mu\left[(w\,(x,y) - 2q)\mathrm{d}p^2 + 2\mathrm{d}p\mathrm{d}q\right] \tag{6}$$

where $\mu = A\Phi + B$ with $A, B \in \mathbb{R}$, $\Phi\,(x,y)$ is a non constant harmonic function, $f = (\nabla\Phi)^2\sqrt{|\mu|}/\mu$, and $w\,(x,y)$ is a solution of the **Euler-Darboux-Poisson equation**

$$\Delta w + (\partial_x \ln|\mu|)\,\partial_x w + (\partial_y \ln|\mu|)\,\partial_y w = \rho$$

where Δ is the Laplace operator in the (x,y) −plane and $T_{\mu\nu} = \rho\delta_{\mu 3}\delta_{\nu 3}$ is the energy-momentum tensor.

It is invariant for the non Abelian Lie agebra $\mathcal{G}_2$ of **Killing fields**, generated by

$$X = \frac{\partial}{\partial p}, \qquad Y = \exp(p)\,\frac{\partial}{\partial q}$$

with $[X,Y] = Y,\;\; g\,(Y,Y) = 0$ and whose orthogonal distribution is integrable.

In the particular case $s = 1$, $f = 1/2$ and $\mu = 1$, the above metrics are locally diffeomorphic [7] to a subclass of the vacuum Peres solutions [16,23] and, by using the transformation

$$p = \ln|u|\,, \qquad q = uv$$

can be written in the form

$$g = \mathrm{d}x^2 + \mathrm{d}y^2 + 2\mathrm{d}u\mathrm{d}v + \frac{w}{u^2}\mathrm{d}u^2. \tag{7}$$

The above metric is of the **Kerr-Schild form**

$$g_{\mu\nu} = \eta_{\mu\nu} + Vk_\mu k_\nu, \qquad k_\mu k^\mu = 0$$

and represents a perturbation of Minkowski metric $\eta = \mathrm{d}x^2 + \mathrm{d}y^2 + 2\mathrm{d}u\mathrm{d}v = \mathrm{d}x^2+\mathrm{d}y^2+\mathrm{d}z^2-\mathrm{d}t^2$, where $u = (z-t)/\sqrt{2}\quad v = (z+t)/\sqrt{2}$, with perturbation given by

$$h := h_{00} = h_{33} = -h_{03} = -h_{30} = \frac{w}{(z-t)^2}\,\cdot$$

Therefore we have

$$\mathbf{E}_{(g)} = -\frac{1}{4}(w_x, w_y, \frac{w}{u})u^{-2}, \qquad \mathbf{B}_{(g)} = \frac{1}{4}(w_y, -w_x, \frac{w}{u})u^{-2}.$$

Thus, gravitational force acting over a massles particle is given by

$$\mathbf{f}_{(g)} = -[w_x(1-v_z)\mathbf{i} + w_y(1-v_z)\mathbf{j} + (w_xv_x + w_yv_y)\mathbf{k}]/4u^2.$$

The velocity of photons is determined by the null geodesics equation

$$(h-1) - 2hv_z + (h+1)v_z^2 = 0$$

which has two solutions

$$v_z = 1, \qquad v_z = \frac{h-1}{h+1} = \frac{w-u^2}{w+u^2}.$$

If the photon propagates parallel to the light beam, $v = (0,0,1)$, then

$$\mathbf{f}_{(g)} = 0$$

and there is not attraction or repulsion.

If the photon propagates antiparallel to the light beam $v = (0,\,0,\,(h-1)/(h+1))$ with

$$\mathbf{f}_{(g)} = -\nabla w/2\left(w+u^2\right)$$

the force turns out to be attractive.

4. Physical Properties

4.1. Wave Character

The wave character and the polarization of these gravitational fields can be analyzed in many ways. For example, we could use the Zel'manov criterion [28] to show that these are gravitational waves and the Landau-Lifshitz pseudo-tensor to find their propagation direction [5, 6]. However, the algebraic Pirani criterion is easier to handle since it determines the wave character of the solutions and the propagation direction both at once. Moreover, it has been shown that, in the vacuum case, the two methods agree [6]. To use this criterion the Weyl scalars must be evaluated according to the Petrov-Penrose classification [15, 17].

To perform the Petrov-Penrose classification, one has to choose a **tetrad** basis with two real null vector fields and two real spacelike (or two complex null) vector fields. Then, according to the Pirani's criterion, if the metric belongs to type **N** of the Petrov classification, it is a gravitational wave propagating along one of the two real null vector fields. Since ∂_u and ∂_v are null real vector fields and ∂_x and ∂_y are spacelike real vector fields, the above set of coordinates is the right one to apply for the Pirani's criterion.

Since the only nonvanishing components of the Riemann tensor, corresponding to the metric (7), are

$$R_{iuju} = -\partial^2_{ij}\partial_u\varphi, \qquad i, j = x, y$$

this gravitational fields belong to **Petrov type N** [8, 28]. Then, according to the Pirani's criterion, the metric (7) does indeed represent a gravitational wave propagating along the null vector field ∂_u.

It is well known that linearized gravitational waves can be characterized entirely in terms of the linearized and gauge invariant Weyl scalars. The non vanishing Weyl scalar of a typical **spin−2 gravitational wave** is Ψ_4. The metrics (7) also have as non vanishing Weyl scalar Ψ_4.

4.2. Spin

A transparent method to determine the spin of a gravitational wave is to look at its physical degrees of freedom, i.e., the components which contribute to the energy [10]. One should use the Landau-Lifshitz (pseudo)-tensor t^μ_ν which, in the asymptotically flat case, agrees with the Bondi flux at infinity [6].

It is worth to remark that the canonical and the Landau Lifchitz energy-momentum pseudo-tensors are tensors for Lorentz transformations. Thus, any Lorentz transformation will preserve the form of these tensor and this allows to perform the analysis according to the Dirac procedure. A globally square integrable solution $h_{\mu\nu}$ of the wave equation is a function of $r = k_\mu x^\mu$ with $k_\mu k^\mu = 0$. With the choice

$k_\mu = (1, 0, 0, -1)$, we get for the energy density t_0^0 and the energy momentum t_0^3 the following result

$$16\pi t_0^0 = \frac{1}{4}\left(u_{11} - u_{22}\right)^2 + u_{12}^2, \qquad t_0^0 = t_0^3$$

where $u_{\mu\nu} \equiv \mathrm{d}h_{\mu\nu}/\mathrm{d}r$. Thus, the physical components which contribute to the energy density are $h_{11} - h_{22}$ and h_{12}. Following the analysis of [10], we see that they are eigenvectors of the infinitesimal rotation generator $\mathcal{R}$, in the plane $x - y$, belonging to the eigenvalues $\pm 2\mathrm{i}$. The components of $h_{\mu\nu}$ which contribute to the energy thus correspond to spin-2.

In the case of the prototype of spin-1 gravitational waves (7), we have

$$\tau_0^0 \sim c_1(h_{0x,x})^2 + c_2(h_{0y,x})^2, \qquad t_0^0 = t_0^3$$

where c_1 e c_2 constants, so that the physical components of the metric are h_{0x} and h_{0y}. Following the previous analysis one can see that these two components are eigenvectors of $\mathrm{i}\mathcal{R}$ belonging to the eigenvalues ± 1. In other words, metrics like (7), which are not pure gauge since the Riemann tensor is not vanishing, represent spin-1 gravitational waves propagating along the $z-$axis at light velocity.

Summarizing: *globally square integrable* spin-1 *gravitational waves propagating on a flat background are always pure gauge.* Spin-1 *gravitational waves which are not globally square integrable are not pure gauge.*

What truly distinguishes spin-1 from spin-2 gravitational waves is the fact that in the spin-1 case the Weyl scalar has a non trivial dependence on the transverse coordinates (x, y) due to the presence of the harmonic function. This could led to observable effects on length scales larger than the *characteristic length scale* where the harmonic function changes significantly. Indeed, the Weyl scalar enters in the geodesic deviation equation implying a non standard deformation of a ring of test particles breaking the invariance under rotation of π around the propagation direction. Eventually, one can say that there should be distinguishable effects of spin-1 waves on suitably large length scales.

It is also worth to stress that the results of [1] suggest that the sources of asymptotically flat PP–waves (which have been interpreted as spin-1 gravitational waves [5, 6]) repel each other. Thus, in a field theoretical perspective, "PP-gravitons" must have spin-1 .

5. Quantum Field Theory

Quantum Field Theory is needed when we confront simultaneously two great physics innovations of the last century of the previous millennium: *special relativity* and *quantum mechanics*. A fast moving rocket ship, close to light velocity, needs special relativity, not quantum mechanics! A slow moving electron scattering on a proton needs quantum mechanics, not special relativity! Particles can come to life and particles can die. It is this matter of birth, life and death that requires the development of a so called quantum field theory. In quantum mechanics the uncertainty principle tells us that energy can fluctuate wildly over a small interval of time. According to special relativity, energy can be converted into mass and viceversa. With quantum mechanics and special relativity, the wildly fluctuating energy can metamorphose into mass, that is in new particles not previously present.

5.1. The Partition Function

It is known from Quantum Field Theory that a consequence of spin-1 messengers is that particles with the same orientation repel and particles with opposite orientation attract. Indeed, path integral formalism describing a massive vector field theory A_μ makes use of the **partition function** defined by

$$Z\left(J\right) =< 0| \exp[-\frac{\mathrm{i}}{\hbar} H\left(J\right) T]|0 >$$

where $\hbar$ is the Planck constant, H the Hamiltonian, J the source and T the interaction time.

It can be represented by using the **Feynman path integral**

$$Z\left(J\right) = \int DA \exp[\frac{\mathrm{i}}{\hbar} S\left(A, J\right)]$$

where

$$S\left(A, J\right) = \int \mathrm{d}^4 x \left(A_\mu \left[\left(\partial^2 + m^2\right) g^{\mu\nu} - \partial^\mu \partial^\nu\right] A_\nu + J^\mu A_\mu\right)$$

is the classical action.

We also have

$$Z\left(J\right) = \exp[\frac{\mathrm{i}}{\hbar} W\left(J\right)]$$

with

$$W\left(J\right) = -\frac{1}{2} \int \mathrm{d}^4 x \mathrm{d}^4 y J^\mu\left(x\right) D_{\mu\nu}\left(x - y\right) J^\nu\left(y\right)$$

where $D_{\nu\lambda}\left(x\right)$ is the Green function defined by

$$\left[\left(\partial^2 + m^2\right) g^{\mu\nu} - \partial^\mu \partial^\nu\right] D_{\nu\lambda}\left(x\right) = \delta^\mu_\lambda \delta^{(4)}\left(x\right).$$

Taking the Fourier transform, we get

$$W\left(J\right) = -\frac{1}{2} \int \mathrm{d}^4 k J^*\left(x\right) D_{\mu\nu}\left(k\right) J^\nu\left(k\right)$$

where

$$D_{\mu\nu}\left(k\right) = \frac{-g_{\mu\nu} + k_\mu k_\nu}{k^2 - m^2}$$

is called the *propagator for the massive vector field* A_μ.

A simple calculation shows that the potential energy between like charges is given by

$$U = \frac{W}{T} = \frac{\exp(-mr)}{4\pi r}$$

so that $\mathrm{d}U/\mathrm{d}r < 0$ and the force between like charges turns out to be repulsive, as we already know from electrodynamics.

Conclusions

Thus, the apparent lacking of attraction found by TEP and Faraoni-Dunse must be ascribed to the linear approximation since, according to our results, photons generate spin-1 gravitational waves and, as a consequence, two photons with same helicity must repel one another. This repulsion turns out to be very weak and cannot be certainly observed in the Laboratory but it could play a relevant at cosmic scale.

Therefore, one may postulate the existence, together with gravitons, of spin-1 gravitophotons and spin-0 graviscalar. Through coupling to fermions, they might give forces depending on the barion number. These fields might give two (or more) Yukawa type terms of different signs [22], corresponding to repulsive gravitophoton exchange and attractive graviscalar exchange (range » $200m$).

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