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A discrete time interactive exclusive random walk of infinitely many particles on one-dimensional lattices

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§ 1. Introduction and theorems

The aim of this paper is to provide a simple model of discrete time interactive exclusive random walk of infinitely many particles (i.m.p.'s) which yields a simple exclusion process after a simple limiting procedure, and then to show that the method of relative entropy is also applicable to the analysis of stationary measures for a random walk of i.m.p.'s such that i.m.p.'s can move simultaneously.

Suppose $\mathscr{X} \equiv \{0, 1\}^{\mathbf{Z}}$ represents the space of all configurations of indistinguishable i.m.p.'s on one dimensional lattices $\mathbf{Z}$. For a given $\eta \equiv (\cdots \eta_{-1}\eta_0\eta_1 \cdots) \in \mathscr{X}$, the site i is regarded to be occupied by a particle if $\eta_i = 1$. Let $\mathscr{E} = \{\mathrm{e}, \bar{\mathrm{e}}\}^{\mathbf{Z}}$. We associate $\omega \equiv (\cdots \omega_{i-1}\omega_i\omega_{i+1} \cdots) \in \mathscr{E}$ with $\eta \in \mathscr{X}$ and consider that the states η_i and η_{i+1} on the edge $(i, i+1)$ are exchangeable [resp., unexchangeable] if $\omega_i = \mathrm{e}$[resp., $\bar{\mathrm{e}}$]. Then we define an exclusive movement of i.m.p.'s on $\mathbf{Z}$ by the mapping $W_\omega: \mathscr{X} \to \mathscr{X}$ defined by $W_\omega(\eta) = (\cdots \eta'_{-1}\eta'_0\eta'_1 \cdots)$ where

$$\begin{cases} \eta'_i\eta'_{i+1} = \eta_{i+1}\eta_i & \text{iff} \quad \omega_{i-1}\omega_i\omega_{i+1} = \bar{\mathrm{e}}\mathrm{e}\bar{\mathrm{e}}, \\ \eta'_i = \eta_i & \text{otherwise}. \end{cases}$$

More intuitively, the movement of each particle of η is defined through ω of $\mathscr{E}$ in such a way that a particle on the site i moves to the site $i+1$ [resp., $i-1$] if and only if $\omega_{i-1}\omega_i\omega_{i+1} = \bar{\mathrm{e}}\mathrm{e}\bar{\mathrm{e}}$ and $\eta_i = 1$, $\eta_{i+1} = 0$ [resp., $\omega_{i-2}\omega_{i-1}\omega_i = \bar{\mathrm{e}}\mathrm{e}\bar{\mathrm{e}}$ and $\eta_{i-1} = 0$, $\eta_i = 1$]. We remark that if $\eta_i = \eta_{i+1}$, there occurs no change of states on the sites i and $i+1$ even if $\omega_{i-1}\omega_i\omega_{i+1} = \bar{\mathrm{e}}\mathrm{e}\bar{\mathrm{e}}$.

Now suppose that the configuration of i.m.p.'s on $\mathbf{Z}$ at time t is η. Let $\vec{e}(\eta, t)$ be a random element which takes the value in $\mathscr{E}$. Then $W_{\vec{e}(\eta,t)}(\eta)$ defines a random configuration of i.m.p.'s at time $t+1$ which comes from η at time t. In the following we treat the case where the distributions $Q_{(\eta,t)}$ of $\vec{e}(\eta, t)$, $\eta \in \mathscr{X}$, $t = 0, 1, \ldots$, are independent of t, and their common distributions Q_η, $\eta \in \mathscr{X}$, are given as follows: For some fixed constants $0 < \alpha < 1$ and $0 < \beta < 1$

$$(1.1)\qquad \begin{aligned} Q_\eta(\omega\in\mathscr{E}: \omega_i = \mathrm{e}\} &= \begin{cases} \alpha & \text{if } \eta_i \neq \eta_{i+1} \\ \beta & \text{if } \eta_i = \eta_{i+1} \end{cases}, \\ Q_n\{\omega\in\mathscr{E}: \omega_i = \bar{\mathrm{e}}\} &= \begin{cases} 1-\alpha & \text{if } \eta_i \neq \eta_{i+1} \\ 1-\beta & \text{if } \eta_i = \eta_{i+1} \end{cases}, \end{aligned}$$

and random variables π_i: $\mathscr{E} \to \{\mathrm{e}, \bar{\mathrm{e}}\}$ defined by $\pi_i(\omega) = \omega_i$, $i\in\mathbf{Z}$, are mutually independent under Q_η. This assumption leads us to define transition probabilities $P(\eta, A)$ from $\eta\in\mathscr{X}$ to a Borel set $A\subset\mathscr{X}$ by

$$P(\eta, A) = \mathrm{Prob}\{W_{\tilde{e}(\eta,t)}(\eta)\in A\} = Q_\eta\{\omega\in\mathscr{E}: W_\omega(\eta)\in A\}.$$

We will denote by $(\mathrm{EP})_{\alpha,\beta}$ the Markov process defined by the above transition probabilities ("EP" stands for the "exclusion process"). In $(\mathrm{EP})_{\alpha,\beta}$ the exchange of η_i and η_{i+1} and that of η_j and η_{j+1} are independent if $|i-j|\geq 3$; and so infinitely many particles on $\mathbf{Z}$ can move simultaneously under $P(\eta, \cdot)$ when $\sum_i\eta_i = \sum_i(1-\eta_i) = \infty$. Moreover if $\alpha = \beta$, then Q_η's are independent of $\eta\in\mathscr{X}$, that is, the exchange of η_i and η_{i+1} is not affected by the configuration of the other sites, and its probability is given by $\bar{\alpha}\alpha\bar{\alpha}$, where $\bar{\alpha} = 1-\alpha$. Thus the Markov process in the title of this paper is obtained.

Up to now various results have been obtained concerning simple exclusion process. (Remember that the simple exclusion process is a continuous time Markov process on $\mathscr{X}$, and the number of particles which jump at the same time is one. For details, see the textbook of Liggett [7]). However there are not so many results about discrete time exclusion process. In this paper we first show that a well-known simple exclusion process is obtained from our discrete time model $(\mathrm{EP})_{\alpha,\beta}$ by a simple limiting procedure. Then we investigate the structure of the set of stationary measures for $(\mathrm{EP})_{\alpha,\beta}$.

Let us denote by $S_{\alpha,\beta}(k)$, $k = 0, 1, \dots$, the semigroup corresponding to $(\mathrm{EP})_{\alpha,\beta}$. Let $\tilde{T}(t)$, $t\in[0, \infty)$ be the semigroup of the simple exclusion process on $\mathbf{Z}$ whose generator is given by

$$(A_\infty g)(\eta) = \textstyle\sum_{l\in\{i,i+1,\dots,j+1\}} \alpha\{g(\eta^{l-1,l}) - g(\eta)\}$$

for a $\mathscr{B}_{i,j}$-measurable function g, where $\mathscr{B}_{i,j}$ is the σ-field generated by $\{\eta_i, \eta_{i+1}, \dots, \eta_j\}$ and $\eta^{l-1,l}$ is the element of $\mathscr{X}$ such that η_{l-1} and η_l are exchanged in η. Let $C(\mathscr{X})$ be the set of all continuous functions on $\mathscr{X}$ (the topology of $\mathscr{X}$ is the product of the discrete topology on $\{0, 1\}$). Then using the theorem of Kurtz [1] we have

THEOREM 1. *For each* $f\in C(\mathscr{X})$,

$$\lim_{n\to\infty}\|S_{\alpha/n,\beta/n}([nt])f - \tilde{T}(t)f\|_\infty = 0 \text{ for all } t\geq 0,$$

where $[r]$ *is the largest integer not exceeding* r.

When one sees the proof of Theorem 1, one will recognize at once the mechanism that, differently from $S_{\alpha/n,\beta/n}$, only one particle can jump at each time in the limit process $\tilde{T}$.

As for the tools for the analysis of stationary measures for exclusion processes we know the method of coupled Markov process and the method of relative entropy. The former is very useful and clear [6, 7] if the movement of each particle is not influenced by the other particles, and is also applicable to our $(\mathrm{EP})_{\alpha,\beta}$ if $\alpha=\beta$. However we employ in this paper the latter method because it can treat the interactive case $\alpha\neq\beta$ as well. This method was used for stochastic Ising models in [4, 5] and for interactive exclusion processes in [3, 8, 10]. Especially in [10], the structure of stationary measures is completely determined. However in our $(\mathrm{EP})_{\alpha,\beta}$, differently from [10], i.m.p.'s can move simultaneously. Hence we can not make use of the argument given in [10] which bases on the property that no more than one particle can move at the same time. So the argument given here, which evaluates the internal entropy in $[-N, N]$, $N\in\mathbf{N}$, by the entropy on the boundaries, is different from that of [10]. Our result about stationary measures for $(\mathrm{EP})_{\alpha,\beta}$ is the following:

THEOREM 2. *A probability measure* ν *on* $\mathscr{X}$ *is stationary for the Markov process* $(\mathrm{EP})_{\alpha,\beta}$ *if and only if* ν *has the regular clustering property* $((\mathrm{RCP})_\gamma)$ *with index* $\gamma\equiv[(1-\alpha)/(1-\beta)]^2$.

The definition of $(\mathrm{RCP})_\gamma$, which is equivalent to the condition that a measure is a canonical Gibbs state in the statistical mechanics, is given in §2. It is shown in [2 + 3, 10] that the extremal points of the set of $(\mathrm{RCP})_\gamma$-measures are $\{\mu_\rho^{(\gamma)}\}_{0\le\rho\le1}$, where $\mu_\rho^{(\gamma)}$ is a renewal measure on $\mathscr{X}$ with $\mu_\rho^{(\gamma)}\{\eta\in\mathscr{X}:\eta_i=1\}=\rho$ (more precisely $\mu_\rho^{(\gamma)}$ is a Gibbs state with nearest neighbor potential $-kT\log\gamma$). For details, see §2. Thus the structure of stationary measures for $(\mathrm{EP})_{\alpha,\beta}$ is completely known. As a corollary of Theorem 2 we have

Corollary. *Every stationary measure* ν *for* $(\mathrm{EP})_{\alpha,\beta}$ *is reversible, that is,*

$$\int fS_{\alpha,\beta}(t)g\,d\nu=\int gS_{\alpha,\beta}(t)f\,d\nu,\ t>0,$$

for every f, $g\in C(\mathscr{X})$.

If we denote by $\mathscr{M}_\gamma$ the totality of $(\mathrm{RCP})_\gamma$-measures, Theorem 2 states that the set of stationary measures for $S_{\alpha/n,\beta/n}(k)$ is $\mathscr{M}_{\gamma_n}$ with $\gamma_n=[(1-\alpha/n)/(1-\beta/n)]^2$. Then Theorem 1 suggests that the set of stationary measures for $\tilde{T}(t)$ will be $\mathscr{M}_1$. This coincides with the well-known fact that a probability measure ν on $\mathscr{X}$ is stationary for the simple exclusion $\tilde{T}(t)$ if and only if it has the exchangeable property, i.e., $\nu \in \mathscr{M}_1$. Thus we think our $(\mathrm{EP})_{\alpha,\beta}$ is an interesting discrete time interactive model which has a close connection with simple exclusion processes, and the structure of whose stationary measures is completely determined.

The proofs of Theorems 1 and 2 are given in §2 by using three lemmas. The proofs of lemmas will be given in §3.

§2. Difinitions and proofs of theorems

In this section we give the proofs of Theorems 1 and 2. We first prepare some notation and definitions. Notation given in the previous section will be used without any comments.

By $\mathscr{C}_{i,j}$, $i \le j$, $i, j \in \mathbf{Z}$, we denote the set of basic cylinders ${}_i[a_i a_{i+1} \cdots a_{j-1} a_j]_j \equiv \{\eta \in \mathscr{X}: \eta_l = a_l,\ i \le l \le j\}$, $a_i \cdots a_j \in \{0, 1\}^{j-i+1}$. Elements of $\mathscr{C}_{i,j}$ are sometimes denoted by $\boldsymbol{a}$, $\boldsymbol{b}$ or $a(i, j)$, $b(i, j)$ and so on. The σ-field $\mathscr{B}_{i,j}$ is generated by $\mathscr{C}_{i,j}$, i.e., $\mathscr{B}_{i,j} = \sigma(\mathscr{C}_{i,j})$. We set $\mathscr{C} = \{\emptyset\} \cup \{\bigcup_{i \le j} \mathscr{C}_{i,j}\}$ and $\mathscr{B} = \sigma(\mathscr{C})$. We endow $\mathscr{E} \equiv \{\mathrm{e}, \bar{\mathrm{e}}\}^{\mathbf{Z}}$ with the Borel structure generated by $\bigcup_{i \le j} \mathscr{F}_{i,j}$, where $\mathscr{F}_{i,j} = \{{}_i[E_i \cdots E_j]_j : E_i \cdots E_j \in \{\mathrm{e}, \bar{\mathrm{e}}\}^{j-i+1}\}$. Elements of $\mathscr{F}_{i,j-1}$ are sometimes denoted by $\boldsymbol{E}$, $\boldsymbol{F}$ or $E(i, j-1)$, $F(i, j-1)$ and so on.

Given $\boldsymbol{E} \equiv {}_i[E_i E_{i-1} \cdots E_{j-1}]_{j-1} \in \mathscr{F}_{i,j-1}$, let $W_{\boldsymbol{E}}: \mathscr{C}_{i,j} \to \mathscr{C}_{i,j}$ be the mapping defined by $W_{\boldsymbol{E}}({}_i[a_i a_{i+1} \cdots a_{j-1} a_j]_j) = {}_i[a_i a'_{i+1} \cdots a'_{j-1} a_j]_j$ where $a'_l a'_{l+1} = a_{l+1} a_l$ if and only if $E_{l-1} E_l E_{l+1} = \bar{\mathrm{e}}\mathrm{e}\bar{\mathrm{e}}$ for l with $i < l < j-1$, and $a'_l = a_l$ otherwise. We note that $W_{\boldsymbol{E}} \circ W_{\boldsymbol{E}}$ is an identity mapping on $\mathscr{C}_{i,j}$ as well as $W_\omega \circ W_\omega$ on $\mathscr{X}$.

In what follows probability measures $Q_\eta(\cdot)$, $\eta \in \mathscr{X}$, will be written by $Q(\eta, \cdot)$. Since for $\boldsymbol{E} \in \mathscr{F}_{i,j-1}$ $Q(\eta, \boldsymbol{E})$ depends only on $\eta_i, \eta_{i+1}, \dots, \eta_j$, we can define $Q(\boldsymbol{a}, \boldsymbol{E})$, $\boldsymbol{a} \in \mathscr{C}_{i,j}$, $\boldsymbol{E} \in \mathscr{F}_{i,j-1}$, to be $Q(\eta, \boldsymbol{E})$ for some $\eta \in \mathscr{X}$ satisfying $\eta \in \boldsymbol{a}$. Probabilities $Q({}_i[a_i \cdots a_j]_j, {}_i[E_i \cdots E_{j-1}]_{j-1})$ will be sometimes abbreviated such as $Q(a_i \cdots a_j, E_i \cdots E_{j-1})$. We note that

$$Q(a_l a_{l+1}, E_l) \in \{\alpha, 1-\alpha, \beta, 1-\beta\}$$

by (1.1), and

$$Q(a_i \cdots a_j, E_i \cdots E_{j-1}) = \textstyle\prod_{l=i}^{j-1} Q(a_l a_{l+1}, E_l)$$

by the mutual independence of $\{\pi_j\}_{i \in \mathbf{Z}}$. We also define $P(b(i-2, j+2), {}_i[a_i \cdots a_j]_j)$ analogously.

PROOF OF THEOREM 1. We use the theorem of Kurtz [1] by taking $D(\mathscr{X})$ in §3 of Chapter I of Liggett [7] as a core for A_∞. Set $A_n = n\{A_{\alpha/n, \beta/n}(1) - I\}$. By definition we have for $\boldsymbol{a} \in \mathscr{C}_{i,j}$

$$P(\eta, \boldsymbol{a}) = \sum_{\boldsymbol{E} \in \mathscr{F}_{i-2,j+1}} Q(\eta, \boldsymbol{E}) \chi_{\boldsymbol{a}}(W_{\boldsymbol{E}}(\eta(i-2, j+2))),$$

where $\eta(i-2, j+2) = {}_{i-2}[\eta_{i-2}\eta_{i-1} \cdots \eta_{j+2}]_{j+2}$ for $\eta \in \mathscr{X}$ and

$$\chi_{\boldsymbol{a}}(W_{\boldsymbol{E}}(\eta(i-2, j+2))) = \begin{cases} 1 & \text{if } W_{\boldsymbol{E}}(\eta(i-2, j+2)) \subset \boldsymbol{a} \\ 0 & \text{otherwise}. \end{cases}$$

Therefore for $\mathscr{B}_{i,j}$-measurable functions g, $A_n g(\eta)$ is represented as the sum of the terms of the form

$$n\left(\frac{\alpha}{n}\right)^k \left(\frac{\beta}{n}\right)^l \left(1 - \frac{\alpha}{n}\right)^u \left(1 - \frac{\beta}{n}\right)^v \{g(W_{\boldsymbol{E}}(\eta(i-2, j+2))) - g(\eta(i-2, j+2))\},$$
$$k + l + u + v = j - i + 4,$$

over $\boldsymbol{E}$ satisfying $W_{\boldsymbol{E}}(\eta(i-2, j+2))) \neq \eta(i-2, j+2)$. This condition on $\boldsymbol{E}$ implies that only terms with $k \geq 1$ and hence only terms with $k = 1$ and $l = 0$ can remain in the limit of $A_n g$, which gives $\lim_{n \to \infty} A_n g = A_\infty g$. Then it is not hard to show that $A_n f \to A_\infty f$ for every $f \in D(\mathscr{X})$. Thus the condition (c) of Theorem 6.5 in [1] is satisfied. □

A probability measure ν on $\mathscr{X}$ is said to have the *regular clustering property* $(\mathrm{RCP})_\gamma$ with index $\gamma > 0$ if it satisfies

$$\gamma^{\#_{01}(\boldsymbol{a})} \nu(\boldsymbol{a}) = \gamma^{\#_{01}(\boldsymbol{b})} \nu(\boldsymbol{b})$$

for all $\boldsymbol{a} = {}_i[a_i \cdots a_j]_j$, $\boldsymbol{b} = {}_i[b_i \cdots b_j]_j \in \mathscr{C}_{i,j}$, $i \leq j$, $i, j \in \mathbf{Z}$, with

$$a_i = b_i, \ a_j = b_j \text{ and } \sum_{k=i}^{j} a_k = \sum_{k=i}^{j} b_k,$$

where

$$\#_{uv}({}_i[a_i \cdots a_j]_j) = \#\{k \colon a_k a_{k+1} = uv, \ i \leq k \leq j-1\}.$$

We remark that $(\mathrm{RCP})_\gamma$ of μ is equivalent to

$$Q(\boldsymbol{a}, \boldsymbol{E})\mu(\boldsymbol{a}) = Q(W_{\boldsymbol{E}}(\boldsymbol{a}), \boldsymbol{E})\mu(W_{\boldsymbol{E}}(\boldsymbol{a})) \tag{2.1}$$

for every $\boldsymbol{a} \in \mathscr{C}_{i,j}$ and $\boldsymbol{E} = \mathscr{F}_{i,j-1}$, $i < j$ (to understand the meaning of (2.1) just consider the simple case $\boldsymbol{E} = {}_i[\bar{\mathrm{e}} \cdots \bar{\mathrm{e}}\mathrm{e}\bar{\mathrm{e}} \cdots \bar{\mathrm{e}}]_{j-1}$). Further this relation essentially relies upon the property that $Q(01, \mathrm{e}) = Q(10, \mathrm{e}) = \alpha$. The reader who is familiar with the statistical mechanics can easily check that $(\mathrm{RCP})_\gamma$ is equivalent to the condition that a measure is a canonical Gibbs state with the nearest neighbor potential $-kT\log\gamma$ ([2, 3]).

It is shown [2 + 3, 10] that the set of extremal points of regular clustering measures with index γ is $\{\mu_\rho^{(\gamma)}\}_{0 \leq \rho \leq 1}$. Here $\mu_\rho^{(\gamma)}$ is the translation invariant probability measure on $\mathscr{X}$ defined by

$$\mu_\rho^{(\gamma)}([1]) = \rho, \quad \mu_\rho^{(\gamma)}([0]) = 1 - \rho,$$

$$\mu_\rho^{(\gamma)}([a_i \cdots a_j 00]) = q\mu_\rho^{(\gamma)}([a_1 \cdots a_j 0]),$$

$$\mu_\rho^{(\gamma)}([a_i \cdots a_j 11]) = q'\mu_\rho^{(\gamma)}([a_i \cdots a_j 1]),$$

where q and q' is a unique pair of numbers in the interval (0, 1) satisfying

$$qq'/[(1-q)(1-q')] = \gamma \quad \text{and } (1-q')/(1-q) = (1-q)/\rho;$$

and $\mu_0^{(\gamma)}$ [resp., $\mu_1^{(\gamma)}$] is the Dirac measure $\delta_{\mathbf{0}}$[resp., $\delta_{\mathbf{1}}$] which concentrates at $\mathbf{0} \equiv (\cdots 000 \cdots)$ [resp., $\mathbf{1} \equiv (\cdots 111 \cdots)$]. It is easy to check that $\mu_\rho^{(\gamma)}$ is a Gibbs state with the nearest neighbor interaction on $\mathbf{Z}$ such that the chemical potential $J_0 \equiv J_0(\gamma, \rho)$ and the interaction potential $J_1 \equiv J_1(\gamma)$ are given by

$$J_0 = kT\{2\log q - \log(1-q) - \log(1-q')\} \quad (= kT\log[\gamma q/q'])$$

and

$$J_1 = -kT\{\log q + \log q' - \log(1-q) - \log(1-q')\} \quad (= -kT\log\gamma),$$

respectively, where k is the Boltzmann constant and T is the absolute temperature. Here a probability measure ν on $\mathscr{X}$ is said to be a Gibbs state on $\mathbf{Z}$ with a chemical potential J_0 and a nearest neighbor interaction potential J_1 if its conditional probability $\nu\{_i[a_i \cdots a_j]_j | \mathscr{B}_{i,j}^c\}(\eta)$ of $_i[a_i \cdots a_j]_j \in \mathscr{C}_{i,j}$ for a given $\mathscr{B}_{i,j}^c$ ($\equiv$ the σ-field generated by $\mathscr{C}_{I,J}$, $I \leq J < i$ and $j < I \leq J$) is equal to

$$\Xi_{i,j}(\eta)^{-1}\exp[-(1/kT)\{J_0\textstyle\sum_{k=i}^j a_k + J_1(\eta_{i-1}a_i + a_j\eta_{j+1} + \sum_{k=i}^{j-1} a_k a_{k+1})\}],$$

where $\Xi_{i,j}(\eta)$ is the normalizing factor.

A probability measure ν on $\mathscr{X}$ is said to be stationary for the Markov process $(\mathrm{EP})_{\alpha,\beta}$ if it satisfies

$$\int_{\mathscr{X}} d\nu(\eta)f(\eta) = \int_{\mathscr{X}} d\nu(\eta)\int_{\mathscr{X}} P(\eta, d\xi)f(\xi)$$

for all bounded $\mathscr{B}$-measurable functions f.

PROOF OF THE SUFFICIENCY PART OF THEOREM 2. The sufficiency of the condition $(\mathrm{RCP})_\gamma$ is almost obvious from (2.1) for ν. Indeed for $\boldsymbol{a} \equiv {}_i[a_i \cdots a_j]_j \in \mathscr{C}_{i,j}$

$$\begin{aligned}\int_{\mathscr{X}} d\nu(\eta)P(\eta, \boldsymbol{a}) &= \textstyle\sum_{\boldsymbol{b}\in\mathscr{C}_{i-2,j+2}} \nu(\boldsymbol{b})P(\boldsymbol{b}, \boldsymbol{a}) \\ &= \textstyle\sum_{\boldsymbol{b}} Q(\boldsymbol{b}, \{F \in \mathscr{F}_{i-2,j+1} : W_F(\boldsymbol{b}) \subset \boldsymbol{a}\})\nu(\boldsymbol{b}) \\ &= \textstyle\sum_{\eta^\ell}\sum_{\eta^r}\sum_{\boldsymbol{b}} Q(\boldsymbol{b}, \{F : W_F(\boldsymbol{b}) = \eta^\ell \boldsymbol{a}\eta^r\})\nu(\boldsymbol{b})\end{aligned}$$

$$= \sum_{\eta^\ell} \sum_{\eta^r} \sum_F \sum_b Q(W_F(b), F)\, \delta_{\eta^\ell a \eta^r}(b)\, \nu(W_F(b))$$

$$[\delta_x(y) = 1 \text{ if } y = x, \; = 0 \text{ otherwise}]$$

$$\begin{aligned}
(2.2) \qquad &= \sum_{\eta^\ell} \sum_{\eta^r} \sum_F Q(W_F(\eta^\ell a \eta^r), F)\nu(W_F(\eta^\ell a \eta^r)) \\
&= \sum_{\eta^\ell} \sum_{\eta^r} \sum_F Q(\eta^\ell a \eta^r, F)\nu(\eta^\ell a \eta^r) \qquad [\text{by } (2.1)] \\
&= \nu(a) \qquad\qquad [\text{by } \textstyle\sum_F Q(b, F) = 1],
\end{aligned}$$

where $\sum_{\eta^\ell}$ and $\sum_{\eta^r}$ are summations on $\{0, 1\}^2$, and

$$\eta^\ell a \eta^r = {}_{i-2}[\eta^\ell]_{i-1} \cap a \cap {}_{j+1}[\eta^r]_{j+2} \qquad \text{for } \eta^\ell,\ \eta^r \in \{0, 1\}^2.$$

This implies the stationarity of ν. $\square$

Now let us begin the proof of the necessity part of Theorem 2. Let $\gamma = [(1-\alpha)/(1-\beta)]^2$ as in Theorem 2. In the following, by μ we represent a fixed nontrivial (RCP) ${}_\gamma$-measure $\mu_\rho^{(\gamma)}$, and by ν an arbitrary probability measure on $\mathscr{X}$. We denote by $\Psi(x)$ the function $x \cdot \log x$, $x \geq 0$, with $\Psi(0) = 0$ as usual. The *relative entropy* of ν with respect to μ on $\{-N, -N+1, \ldots, N-1, N\}$, $N \in \mathbf{N}$, is then defined by

$$H_N(\nu) = \sum_{a \in \mathscr{C}_{-N,N}} \mu(a)\, \Psi\!\left(\frac{\nu(a)}{\mu(a)}\right).$$

Suppose that the distribution of $(\mathrm{EP})_{\alpha,\beta}$ at $t = 0$ is ν. Then the distribution $\tilde{\nu}$ at $t = 1$ is given by $\tilde{\nu}(\cdot) = \int \nu(d\eta)P(\eta, \cdot)$, and hence for $a \in \mathscr{C}_{-N,N}$

$$(2.3) \qquad \tilde{\nu}(a) = \sum_{E \in \mathscr{F}_{-N,N-1}} Q(a, E) R(a, E)$$

by (2.2), where

$$R(a, E) = \sum_{\omega^\ell} \sum_{\omega^r} \sum_{\eta^\ell} \sum_{\eta^r} \frac{Q(W_{\omega^\ell E \omega^r}(\eta^\ell a \eta^r), \omega^\ell E \omega^r)}{Q(a, E)}\, \nu(W_{\omega^\ell E \omega^r}(\eta^\ell a \eta^r))$$

and $\sum_{\omega^\ell}$, $\sum_{\omega^r}$ and $\sum_{\eta^\ell}$, $\sum_{\eta^r}$ are summations on $\{\mathrm{e}, \bar{\mathrm{e}}\}^2$ and $\{0, 1\}^2$ respectively ($\omega^\ell E \omega^r = {}_{-N-2}[\omega^\ell]_{-N-1} \cap E \cap {}_N[\omega^r]_{N+1}$).

Let us introduce

$$F_N(\nu) = H_N(\tilde{\nu}) - \sum_{a \in \mathscr{C}_{-N,N}} \mu(a) \sum_{E \in \mathscr{F}_{-N,N-1}} Q(a, E)\, \Psi\!\left(\frac{R(a, E)}{\mu(a)}\right)$$

in order to estimate the range of $R(a, E)/\mu(a)$ over $E \in \mathscr{F}_{-N,N-1}$. It is immediate from

$$H_N(\tilde{\nu}) = \sum_{a \in \mathscr{C}_{-N,N}} \mu(a)\, \Psi\!\left(\sum_{E \in \mathscr{F}_{-N,N-1}} Q(a, E) \frac{R(a, E)}{\mu(a)}\right)$$

and the convexity of Ψ that

$$F_N(\nu) \leq 0 \quad \text{for every } N. \tag{2.4}$$

The key of the proof of Theorem 2 is to show that $F_N(\nu) = 0$ for all stationary measures ν.

To prove Theorem 2 we prepare three lemmas. Proofs of them will be given in the next section. The next lemma brings us a recursive relation concerning $F_N(\nu)$. Let us write

$$L_N(\nu) = \sum_{a \in \mathscr{C}_{-N,N}} \sum_{E \in \mathscr{F}_{-N,N-1}} Q(\boldsymbol{a}, \boldsymbol{E}) R(\boldsymbol{a}, \boldsymbol{E}) \times \log \left\{ \frac{\sum_{\omega_{-N}\omega_{N-1}} Q(\boldsymbol{a}^{(\ell)}, \omega_{-N}) Q(\boldsymbol{a}^{(r)}, \omega_{N-1}) R(\boldsymbol{a}, \omega_{-N} \boldsymbol{E}' \omega_{N-1})}{R(\boldsymbol{a}, \boldsymbol{E})} \right\}$$

where $\boldsymbol{a}^{(\ell)} = a(-N, -N+1)$, $\boldsymbol{a}^{(r)} = a(N-1, N)$ and $\boldsymbol{E}' = E(-N+1, N-2))$.

LEMMA 1. *Suppose $\nu(\boldsymbol{a}) > 0$ for every nonempty $\boldsymbol{a} \in \mathscr{C}$. Then we have*

$$F_N(\nu) \leq F_{N-1}(\nu) + L_N(\nu), \quad N \geq 2,$$

and $L_N(\nu) \leq 0$.

The next lemma states that if ν is stationary, then $F_N(\nu)$'s are bounded below.

LEMMA 2. *Suppose ν is stationary for* $(\mathrm{EP})_{\alpha,\beta}$ *and $\nu(\boldsymbol{a}) > 0$ for all nonempty $\boldsymbol{a} \in \mathscr{C}$. Then there exists a positive constant c such that $F_N(\nu) \geq -c$ for all N.*

Then, combining Lemmas 1 and 2, we can prove

LEMMA 3. *Suppose ν is stationary for* $(\mathrm{EP})_{\alpha,\beta}$ *and $\nu(\boldsymbol{a}) > 0$ for all nonempty $\boldsymbol{a} \in \mathscr{C}$. Then $\lim_{N \to \infty} F_N(\nu) = 0$, and hence $F_N(\nu) = 0$ for all $N \in \mathbf{N}$.*

With these Lemmas we can complete the proof of Theorem 2.

PROOF OF THE NECESSITY PART OF THEOREM 2. It is clear that $\delta_{\mathbf{0}}$ and $\delta_{\mathbf{1}}$ are stationary measures and have $(\mathrm{RCP})_\gamma$. Hence it is sufficient for the proof of the necessity to show that every stationary measure satisfying $\nu(\{\mathbf{0}, \mathbf{1}\}) = 0$ has $(\mathrm{RCP})_\gamma$. Just as in the proof of Proposition 4.1 in [10] we can show that $\nu(\boldsymbol{a}) > 0$ for all nonempty $\boldsymbol{a} \in \mathscr{C}$. By Lemma 3 we have $F_N(\nu) = 0$ for every N. Since $\Psi(x)$ is strictly convex, this implies that if we fix N then for each $\boldsymbol{a} \in \mathscr{C}_{-N,N}$ there exists a constant $\Gamma(\boldsymbol{a})$ such that $R(\boldsymbol{a}, \boldsymbol{E}) = \Gamma(\boldsymbol{a})$ for every $\boldsymbol{E} \in \mathscr{F}_{-N,N-1}$. From (2.3) and $\tilde{\nu} = \nu$ it is known that $\Gamma(\boldsymbol{a}) = \nu(\boldsymbol{a})$ and so

$$R(\boldsymbol{a}, \boldsymbol{E}) = \nu(\boldsymbol{a}) \quad \text{for all } \boldsymbol{E} \in \mathscr{F}_{-N,N-1}. \tag{2.5}$$

Further if $\boldsymbol{E}$ is taken to be $E_{i-1}E_iE_{i+1} = \bar{\mathrm{e}}\mathrm{e}\bar{\mathrm{e}}$ and $E_l = \mathrm{e}$ for the others $(-N$

$+3 \leq i \leq N-4$), then by the definition of $R(\boldsymbol{a}, \boldsymbol{E})$

$$R(\boldsymbol{a}, \boldsymbol{E}) = \frac{Q(a_{i-1}a_{i+1}a_ia_{i+2}, \bar{e}e\bar{e})}{Q(a_{i-1}a_ia_{i+1}a_{i+2}, \bar{e}e\bar{e})} \nu({}_{-N}[a_{-N} \cdots a_{i-1}a_{i+1}a_ia_{i+2} \cdots a_N]_N),$$

which together with (2.5) implies $(\mathrm{RCP})_\gamma$ of ν. □

PROOF OF COROLLARY. For the proof it is sufficient to check the case $f = \chi_{\boldsymbol{a}}$ and $g = \chi_{\boldsymbol{b}}$, $\boldsymbol{a}, \boldsymbol{b} \in \mathscr{C}$. But this is almost immediate from (2.1). In fact, suppose $\boldsymbol{a} \in \mathscr{C}_{i,j}$, $\boldsymbol{b} \in \mathscr{C}_{I,J}$ and $i < I < j < J$. Then

$$\begin{aligned}
\int_{\mathscr{X}} fS_{\alpha,\beta}(1)g d\nu &= \int_{\mathscr{X} \cap \boldsymbol{a}} P(\eta, \boldsymbol{b}) d\nu(\eta) \\
&= \textstyle\sum_{\eta^\ell}\sum_{\eta^r} Q(\eta^\ell \boldsymbol{a}\eta^r, \{F \in \mathscr{F}_{i-2,J+1} : W_F(\eta^\ell \boldsymbol{a}\eta^r) \subset \boldsymbol{b}\})\nu(\eta^\ell \boldsymbol{a}\eta^r) \\
&= \textstyle\sum_{\eta^\ell}\sum_{\eta^r}\sum_{\xi^\ell}\sum_{\xi^r}\sum_F Q(\eta^\ell \boldsymbol{a}\eta^r, F)\nu(\eta^\ell \boldsymbol{a}\eta^r)\delta_{\eta^\ell \boldsymbol{a}\eta^r}(W_F(\xi^\ell \boldsymbol{b}\xi^r)) \\
&\qquad\qquad\qquad\qquad\qquad\qquad [\text{by } W_F \circ W_F(\boldsymbol{c}) = \boldsymbol{c}] \\
&= \textstyle\sum_{\eta^\ell}\sum_{\eta^r}\sum_{\xi^\ell}\sum_{\xi^r}\sum_F Q(W_F(\xi^\ell \boldsymbol{b}\xi^r), F)\nu(W_F(\xi^\ell \boldsymbol{b}\xi^r))\delta_{\eta^\ell \boldsymbol{a}\eta^r}(W_F(\xi^\ell \boldsymbol{b}\xi^r)) \\
&= \textstyle\sum_{\eta^\ell}\sum_{\eta^r}\sum_{\xi^\ell}\sum_{\xi^r}\sum_F Q(\xi^\ell \boldsymbol{b}\xi^r, F)\nu(\xi^\ell \boldsymbol{b}^r\xi^r)\delta_{\eta^\ell \boldsymbol{a}\eta^r}(W_F(\xi^\ell \boldsymbol{b}\xi^r)) \quad [\text{by } (2.1)] \\
&= \int_{\mathscr{X} \cap \boldsymbol{b}} P(\eta, \boldsymbol{a}) d\nu(\eta) = \int_{\mathscr{X}} gS_{\alpha,\beta}(1) f d\nu,
\end{aligned}$$

where $\sum_{\eta^\ell}$, $\sum_{\eta^r}$, $\sum_{\xi^\ell}$ and $\sum_{\xi^r}$ are summations on the set $\{0, 1\}^2$. □

§3. Proofs of lemmas

In this section we give the proofs of lemmas which are used in the preceding section.

PROOF OF LEMMA 1. We have

$$\begin{aligned}
F_N(\nu) &= \textstyle\sum_{\boldsymbol{a}}\sum_{\boldsymbol{E}} Q(\boldsymbol{a}, \boldsymbol{E})R(\boldsymbol{a}, \boldsymbol{E}) \log\left\{\dfrac{\sum_{\tilde{\boldsymbol{E}}} Q(\boldsymbol{a}, \tilde{\boldsymbol{E}})R(\boldsymbol{a}, \tilde{\boldsymbol{E}})}{R(\boldsymbol{a}, \boldsymbol{E})}\right\} \\
&= \textstyle\sum_{\boldsymbol{a}}\sum_{\boldsymbol{E}} Q(\boldsymbol{a}, \boldsymbol{E})R(\boldsymbol{a}, \boldsymbol{E}) \\
&\qquad \times \log\left\{\dfrac{\sum_{\tilde{\boldsymbol{E}}} Q(\boldsymbol{a}, \tilde{\boldsymbol{E}})R(\boldsymbol{a}, \tilde{\boldsymbol{E}})}{\sum_{\hat{\omega}_{-N}\hat{\omega}_{N-1}} Q(\boldsymbol{a}^{(\ell)}, \hat{\omega}_{-N})Q(\boldsymbol{a}^{(r)}, \hat{\omega}_{N-1})R(\boldsymbol{a}, \hat{\omega}_{-N}\boldsymbol{E}'\hat{\omega}_{N-1})}\right\} \\
&\qquad + L_N(\nu),
\end{aligned}$$

where $\sum_{\boldsymbol{E}}$ and $\sum_{\tilde{\boldsymbol{E}}}$ are summations on $\mathscr{F}_{-N,N-1}$, and $\sum_{\hat{\omega}_{-N}\hat{\omega}_{N-1}}$ on $\{0, 1\}^2$. Then the first term on the right-hand side is less than or equal to

$$(3.1)\quad \sum_{b\in\mathscr{C}_{-N+1,N-1}}\sum_{E\in\mathscr{F}_{-N+1,N-2}} Q(b, F)$$
$$\times \{\sum_{a_{-N}a_N}\sum_{\hat{\omega}_{-N}\hat{\omega}_{N-1}} Q(u, \hat{\omega}_{-N})Q(v, \hat{\omega}_{N-1})R(a_{-N}ba_N, \hat{\omega}_{-N}F\hat{\omega}_{N-1})\}$$
$$\times \log\left\{\frac{\sum_{a_{-N}a_N}\sum_{\tilde{E}} Q(a_{-N}ba_N, \tilde{E})R(a_{-N}ba_N, \tilde{E})}{\sum_{a_{-N}a_N}\sum_{\hat{\omega}_{-N}\hat{\omega}_{N-1}} Q(u, \hat{\omega}_{-N})Q(v, \hat{\omega}_{N-1})R(a_{-N}ba_N, \hat{\omega}_{-N}F\hat{\omega}_{N-1})}\right\},$$

where $u = a_{-N}b_{-N+1}$ and $v = b_{N-1}a_N$; because

$$a\log(b/a) + c\log(d/c) \leq (a+c)\log[(b+d)/(a+c)] \quad (a, b, c, d > 0)$$

which follows from the concavity of $\log x$. It is easy to check that the denominator [resp., numerator] in the log function in (3.1) is equal to $R(b, F)$ [resp., $\sum_{\tilde{F}} Q(b, \tilde{F})R(b, \tilde{F})$, $\tilde{F}\in\mathscr{F}_{-N+1,N-2}$]. Thus we have $F_N(v) \leq F_{N-1}(v) + L_N(v)$. The inequality $L_N(v) \leq 0$ is an immediate consequence of the convexity of $\Psi(x)$ and $\sum_{\omega_{-N}\omega_{N-1}} Q(a^{(\ell)}, \omega_{-N})(a^{(r)}, \omega_{N-1}) = 1$. □

In the rest of this section, for a given $a\in\mathscr{C}_{-N,N}$, $N\in\mathbf{N}$, we denote by a', a'' and a''' the elements of $\mathscr{C}_{-N+i,N-i}$, $i = 1$, 2 and 3 respectively such that the configuration from $-N+i$ to $N-i$ equals that of a. We also use E', E'' and E''' for $E\in\mathscr{F}_{-N,N-1}$, $N\in\mathbf{N}$, just as a', a'' and a''' for a. Hence, for example, we use the notation $E = E_{-N}E'E_{N-1} = E_{-N}E_{-N+1}E''E_{N-2}E_{N-1}$ and so on. Given $E\in\mathscr{F}_{-N-2,N+1}$ and $a\in\mathscr{C}_{-N-2,N+2}$, we denote by $W_E^{(N)}(a)$ the basic cylinder in $\mathscr{C}_{-N,N}$ such that the configuration from $-N$ to N is equal to that of $W_E(a)$.

PROOF OF LEMMA 2. Writing

$$H_N(v) = \sum_{E\in\mathscr{F}_{-N-2,N+1}}\sum_{a\in\mathscr{C}_{-N-2,N+2}} Q(a, E)v(a)\log\left(\frac{v(a'')}{\mu(a'')}\right)$$
$$= \sum_E\sum_a Q(W_E(a), E)v(W_E(a))\log\left\{\frac{v(W_E^{(N)}(a))}{\mu(W_E^{(N)}(a))}\right\}$$

and, using $\tilde{v} = v$, we have

$$F_N(v) = \sum_E\sum_a Q(W_E(a), E)v(W_E(a))\log\left\{\frac{v(W_E^{(N)}(a))}{\mu(W_E^{(N)}(a))}\cdot\frac{\mu(a'')}{R(a'', E'')}\right\}$$
$$= \sum_E\sum_a Q(a, E)R(a'', E'')\frac{v(W_E(a))}{v(W_E^{(N)}(a))}\cdot\frac{\mu(W_E^{(N)}(a))}{\mu(W_E(a))}\cdot\frac{\mu(a)}{\mu(a'')}$$
$$\times\Psi\left(\frac{v(W_E^{(N)}(a))}{\mu(W_E^{(N)}(a))}\cdot\frac{\mu(a'')}{R(a'', E'')}\right)$$

by (2.1). Then the lemma follows immediately from the facts that $\Psi(u) \geq$

$-e^{-1}$ for $u \geq 0$, fractions outside of Ψ-function are bounded, and $\sum_{E} \sum_{a} Q(a, E) R(a'', E'') = 2^4$ by (2.3). □

PROOF OF LEMMA 3. 1°. By Lemmas 1 and 2 we have $\lim_{N \to \infty} L_N(v) = 0$. Let us write

$$
\begin{aligned}
(3.2) \qquad L_N(v) = &\sum_{a \in \mathscr{C}_{-N,N}} \sum_{F \in \mathscr{F}_{-N+1,N-2}} c_0 Q(W_F(a'), F) v(W_F^{(N-3)}(a')) \\
&\times [\Psi(\textstyle\sum_{\omega_{-N}\omega_{N-1}} Q(a^{(\ell)}, \omega_{-N}) Q(a^{(r)}, \omega_{N-1}) u_{\omega_{-N}\omega_{N-1}}) \\
&- \textstyle\sum_{\omega_{-N}\omega_{N-1}} Q(a^{(\ell)}, \omega_{-N}) Q(a^{(r)}, \omega_{N-1}) \Psi(u_{\omega_{-N}\omega_{N-1}})],
\end{aligned}
$$

where

$$
\begin{aligned}
u_{F_{-N}F_{N-1}} &\equiv u_{F_{-N}F_{N-1}}(a, F) \\
&= \left\{ c_0 v(W_F^{(N-3)}(a')) \frac{Q(W_F(a'), F)}{Q(a', F)} \right\}^{-1} R(a, F_{-N} F F_{N-1}) \\
(3.3) \qquad &\equiv N(a', F) R(a, F_{-N} F F_{N-1}),
\end{aligned}
$$

$$
c_0 = \max \left\{ \frac{Q(b''', G''')}{Q(b'', G'')} \cdot \frac{\max \{ Q(W_G^{(N)}(b), G''), Q(W_{G''}(b''), G'') \}}{Q(W_{G'''}(b'''), G''')} \;\middle|\; b \in \mathscr{C}_{-N-2,N+2},\ G \in \mathscr{F}_{-N-2,N+1},\ N \in \mathbf{N} \right\}.
$$

Here we have introduced $u_{F_{-N}F_{N+1}}$'s so that they are in [0, 1] (note that if $\alpha = \beta$ then $c_0 = 1$). It is elementary from $\lim_{N \to \infty}(3.2) = 0$ to conclude that the limit of

(3.4)
$$
\sum_a \sum_F c_0 Q(W_F(a'), F) v(W_F^{(N-3)}(a')) \sum_{\tilde{\omega}_{-N}\tilde{\omega}_{N-1}} \sum_{\hat{\omega}_{-N}\hat{\omega}_{N-1}} |u_{\tilde{\omega}_{-N}\tilde{\omega}_{N-1}} - u_{\hat{\omega}_{-N}\hat{\omega}_{N-1}}|
$$

is zero, that is,

(3.5)
$$
\lim_{N \to \infty} \sum_a \sum_F \sum_{\tilde{\omega}_{-N}\tilde{\omega}_{N-1}} \sum_{\hat{\omega}_{-N}\hat{\omega}_{N-1}} |R^*(a, \tilde{\omega}_{-N} F \tilde{\omega}_{N-1}) - R^*(a, \hat{\omega}_{-N} F \hat{\omega}_{N-1})| = 0,
$$

where

$$
R^*(a, \omega_{-N} F \omega_{N-1}) = Q(a', F) R(a, \omega_{-N} F \omega_{N-1}).
$$

In fact for real numbers $u_s \in [0, 1]$, $s = 1, 2, 3, 4$, define

$$
\begin{aligned}
\psi_{\theta_l\theta_r}(u_1, u_2, u_3, u_4) = &\Psi(\theta_l\theta_r u_1 + \theta_l\bar{\theta}_r u_2 + \bar{\theta}_l\theta_r u_3 + \bar{\theta}_l\bar{\theta}_r u_4) \\
&- [\theta_l\theta_r \Psi(u_1) + \theta_l\bar{\theta}_r \Psi(u_2) + \bar{\theta}_l\theta_r \Psi(u_3) + \bar{\theta}_l\bar{\theta}_r \Psi(u_4)],
\end{aligned}
$$

$$\varphi(u_1, u_2, u_3, u_4) = \sum_{s,s'=1}^{4} |u_s - u_{s'}|,$$

where θ_l and θ_r vary over α and β, and $\bar{\theta}_l = (1-\theta_l)$, $\bar{\theta}_r = (1-\theta_r)$. If we set for $\varepsilon > 0$

$$D_{\theta_l\theta_r,\varepsilon} = \{(u_1, u_2, u_3, u_4): \psi_{\theta_l\theta_r}(u_1, u_2, u_3, u_4) \le -\varepsilon\} \text{ and}$$

$$D^*_{\theta_l\theta_r,\varepsilon} = \{(u_1, u_2, u_3, u_4): \psi_{\theta_l\theta_r}(u_1, u_2, u_3, u_4) > -\varepsilon\},$$

we can choose constants Γ_ε and $\delta^*(\varepsilon)$ such that $\delta^*(\varepsilon)\downarrow 0$ as $\varepsilon\downarrow 0$ and

$$\left|\frac{\varphi(u_1, u_2, u_3, u_4)}{\psi_{\theta_l\theta_r}(u_1, u_2, u_3, u_4)}\right| \le \Gamma_\varepsilon \text{ for every } (u_1, u_2, u_3, u_4) \in D_{\theta_l\theta_r,\varepsilon},$$

$$\varphi(u_1, u_2, u_3, u_4) \le \delta^*(\varepsilon) \text{ for every } (u_1, u_2, u_3, u_4) \in D^*_{\theta_l\theta_r,\varepsilon}.$$

Then dividing the summation $\sum_{\boldsymbol{a}}\sum_{\boldsymbol{F}}$ in (3.4) into

$$D_{\theta_l\theta_r,\varepsilon,N} = \{(\boldsymbol{a}, \boldsymbol{F}): Q(\boldsymbol{a}^{(\ell)}, \mathrm{e}) = \theta_l,\ Q(\boldsymbol{a}^{(r)}, \mathrm{e}) = \theta_r \text{ and}$$
$$\psi_{\theta_l\theta_r}(u_{\mathrm{ee}}, u_{\mathrm{e}\bar{\mathrm{e}}}, u_{\bar{\mathrm{e}}\mathrm{e}}, u_{\bar{\mathrm{e}}\bar{\mathrm{e}}}) \le -\varepsilon\},$$

$$D^*_{\theta_l\theta_r,\varepsilon,N} = \{(\boldsymbol{a}, \boldsymbol{F}): Q(\boldsymbol{a}^{(\ell)}, \mathrm{e}) = \theta_l,\ Q(\boldsymbol{a}^{(r)}, \mathrm{e}) = \theta_r \text{ and}$$
$$\psi_{\theta_l\theta_r}(u_{\mathrm{ee}}, u_{\mathrm{e}\bar{\mathrm{e}}}, u_{\bar{\mathrm{e}}\mathrm{e}}, u_{\bar{\mathrm{e}}\bar{\mathrm{e}}}) > -\varepsilon\},$$

(note that θ_l and θ_r vary over α and β), we have

$$(3.4) \le \Gamma_\varepsilon |L_N(\nu)| + \delta^*(\varepsilon) c_0 \sum_{\boldsymbol{a}}\sum_{\boldsymbol{F}} Q(W_{\boldsymbol{F}}(\boldsymbol{a}'), \boldsymbol{F}) \nu(W_{\boldsymbol{F}}^{(N-3)}(\boldsymbol{a}')).$$

Hence we obtain (3.5) by taking $\limsup_{N\to\infty}$ in the both sides and letting $\varepsilon\downarrow 0$.

2°. For $\boldsymbol{a} \in \mathscr{C}_{-N,N}$ and $\boldsymbol{E} \in \mathscr{F}_{-N,N-1}$ put

$$\check{\nu}(W_{\boldsymbol{E}}(\boldsymbol{a})) = Q(W_{\boldsymbol{E}}(\boldsymbol{a}), \boldsymbol{E})\nu(W_{\boldsymbol{E}}(\boldsymbol{a})) \text{ and } \check{R}(\boldsymbol{a}, \boldsymbol{E}) = Q(\boldsymbol{a}, \boldsymbol{E})R(\boldsymbol{a}, \boldsymbol{E}).$$

By the stationarity of ν we have $H_N(\check{\nu}) = H_N(\nu)$, and so using (2.1)

$$\begin{aligned} F_N(\nu) &= \sum_{\boldsymbol{a}}\sum_{\boldsymbol{E}} Q(\boldsymbol{a}, \boldsymbol{E})\mu(\boldsymbol{a})\left[\Psi\left(\frac{v_{E_{-N}E_{N-1}}}{\mu(\boldsymbol{a})N(\boldsymbol{a}', \boldsymbol{E}')}\right) - \Psi\left(\frac{u_{E_{-N}E_{N-1}}}{\mu(\boldsymbol{a})N(\boldsymbol{a}', \boldsymbol{E}')}\right)\right] \\ &= \sum_{\boldsymbol{a}}\sum_{\boldsymbol{E}} \frac{Q(\boldsymbol{a}, \boldsymbol{E})}{N(\boldsymbol{a}', \boldsymbol{E}')}[\Psi(v_{E_{-N}E_{N-1}}) - \Psi(u_{E_{-N}E_{N-1}})] \\ &\quad + \sum_{\boldsymbol{a}}\sum_{\boldsymbol{E}} \frac{Q(\boldsymbol{a}, \boldsymbol{E})}{N(\boldsymbol{a}', \boldsymbol{E}')}[v_{E_{-N}E_{N-1}} - u_{E_{-N}E_{N-1}}] \log\frac{1}{\mu(\boldsymbol{a})N(\boldsymbol{a}', \boldsymbol{E}')} \\ &\equiv F_N^{(1)}(\nu) + F_N^{(2)}(\nu), \end{aligned}$$

where $u_{E_{-N}E_{N-1}} = u_{E_{-N}E_{N-1}}(\boldsymbol{a}, \boldsymbol{E}')$,

$$v_{E_{-N}E_{N-1}} \equiv v_{E_{-N}E_{N-1}}(\boldsymbol{a}, \boldsymbol{E}') = \check{\nu}(W_{E_{-N}\boldsymbol{E}'E_{N-1}}(\boldsymbol{a})) \frac{N(\boldsymbol{a}', \boldsymbol{E}')}{Q(\boldsymbol{a}, E_{-N}\boldsymbol{E}'E_{N-1})}$$

and $N(\boldsymbol{a}', \boldsymbol{E}')$ is the one defined in (3.3). By performing a computation like (3.9) below, we can check that for every $\mathscr{C}_{-N+2,N-2} \times \mathscr{F}_{-N+1,N-2}$-measurable function G

$$\begin{aligned}
&\sum_{\boldsymbol{a}} \sum_{\boldsymbol{E}} \frac{Q(\boldsymbol{a}, \boldsymbol{E})}{N(\boldsymbol{a}', \boldsymbol{E}')} [v_{E_{-N}E_{N-1}} - u_{E_{-N}E_{N-1}}] G(a(-N+2, N-2), \boldsymbol{E}') \\
&\quad = \sum_{\boldsymbol{c}} \sum_{\boldsymbol{F}} \{\sum_{\eta^l} \sum_{\eta^r} \sum_{\omega_{-N}\omega_{N-1}} [\check{\nu}(W_{\boldsymbol{E}}(\boldsymbol{a})) - \check{R}(\boldsymbol{a}, \boldsymbol{E})]\} G(\boldsymbol{c}, \boldsymbol{F}) \\
&\quad = \sum_{\boldsymbol{c}} \sum_{\boldsymbol{F}} 0 \cdot G(\boldsymbol{c}, \boldsymbol{F}) = 0,
\end{aligned}$$

where $\sum_{\boldsymbol{c}}$ and $\sum_{\boldsymbol{F}}$ are summations on $\mathscr{C}_{-N+2,N-2}$ and $\mathscr{F}_{-N+1,N-2}$ resectively, and $\boldsymbol{a} = \eta^{\ell} \boldsymbol{c} \eta^{\boldsymbol{r}}$, $\boldsymbol{E} = \omega_{-N}\boldsymbol{F}\omega_{N-1}$. Therefore we can replace $\nu(W_{\boldsymbol{E}'}^{(N-3)}(\boldsymbol{a}'))$ in $\log N(\boldsymbol{a}', \boldsymbol{E}')$ of $F_N^{(2)}(\nu)$ by $\mu(W_{\boldsymbol{E}''}(\boldsymbol{a}''))$. Thus we have

$$F_N^{(2)}(\nu) = \sum_{\boldsymbol{a}} \sum_{\boldsymbol{E}} [\check{\nu}(W_{\boldsymbol{E}}(\boldsymbol{a})) - \check{R}(\boldsymbol{a}, \boldsymbol{E})] \log \left\{ c_0 \frac{\mu(\boldsymbol{a}')}{\mu(\boldsymbol{a})} \cdot \frac{\mu(W_{\boldsymbol{E}''}(\boldsymbol{a}''))}{\mu(W_{\boldsymbol{E}'}(\boldsymbol{a}'))} \right\}. \tag{3.6}$$

3° Let us show that (3.5) gives

$$\lim_{N\to\infty} \sum_{\boldsymbol{a}\in\mathscr{C}_{-N,N}} \sum_{\boldsymbol{E}\in\mathscr{F}_{-N,N-1}} |\check{\nu}(W_{\boldsymbol{E}}(\boldsymbol{a})) - \check{R}(\boldsymbol{a}, \boldsymbol{E})]| = 0. \tag{3.7}$$

To see this it suffices to check that the limit of

$$D_N(\varepsilon^{\ell}, \varepsilon^{\boldsymbol{r}}) \equiv \sum_{\boldsymbol{a}\in\mathscr{C}_{-N,N}} \sum_{\boldsymbol{G}\in\mathscr{F}_{-N+2,N-3}} |\check{\nu}(W_{\varepsilon^{\ell}\boldsymbol{G}\varepsilon^{\boldsymbol{r}}}(\boldsymbol{a})) - \check{R}(\boldsymbol{a}, \varepsilon^{\ell}\boldsymbol{G}\varepsilon^{\boldsymbol{r}})|$$

is zero every fixed ε^{ℓ}, $\varepsilon^{\boldsymbol{r}} \in \{\mathrm{e}, \bar{\mathrm{e}}\}^2$. If $\varepsilon^{\ell} = \varepsilon^{\boldsymbol{r}} = \mathrm{ee}$, this is true because $\check{R}(\boldsymbol{a}, \mathrm{ee}\boldsymbol{G}\mathrm{ee})$ equals $\check{\nu}(W_{\mathrm{ee}\boldsymbol{G}\mathrm{ee}}(\boldsymbol{a}))$. If one of ε^{ℓ} and $\varepsilon^{\boldsymbol{r}}$ is ee, the story is rather simple. We first treat the case $\varepsilon^{\ell} = \mathrm{ee}$.

By (3.5), for every fixed $\tilde{\omega}_{-N}$, $\tilde{\omega}_{-N}$, $\hat{\omega}_{-N}$, $\hat{\omega}_{N-1} \in \{\mathrm{e}, \bar{\mathrm{e}}\}$ and $\omega_{-N+1}\omega_{-N+2}$, $\omega_{N-3}\omega_{N-2} \in \{\mathrm{e}, \bar{\mathrm{e}}\}^2$, we have

$$\begin{aligned}
&\lim_{N\to\infty} \sum_{\boldsymbol{a}\in\mathscr{C}_{-N,N}} \sum_{\boldsymbol{H}\in\mathscr{F}_{-N+3,N-4}} |R^*(\boldsymbol{a}, \tilde{\omega}_{-N}\omega_{-N+1}\omega_{-N+2}\boldsymbol{H}\omega_{N-3}\omega_{N-2}\tilde{\omega}_{N-1}) \\
&\qquad\qquad - R^*(\boldsymbol{a}, \hat{\omega}_{-N}\omega_{-N+1}\omega_{-N+2}\boldsymbol{H}\omega_{N-2}\omega_{N-1}\hat{\omega}_{N-1})| \\
&\equiv \lim_{N\to\infty} S_N(\tilde{\omega}_{-N}\omega_{-N+1}\omega_{-N+2}, \omega_{N-3}\omega_{N-2}\tilde{\omega}_{N-1}; \\
&\qquad\qquad \hat{\omega}_{-N}\omega_{-N+1}\omega_{-N+2}, \omega_{N-3}\omega_{N-2}\hat{\omega}_{N-1}) = 0.
\end{aligned} \tag{3.8}$$

Below we give a computation for $S_N(\mathrm{ee}\omega_{-N+2}, \omega_{N-3}\bar{\mathrm{e}}\bar{\mathrm{e}}; \mathrm{ee}\omega_{-N+2}, \omega_{N-3}\bar{\mathrm{e}}\mathrm{e})$ as an example (this kind of computation will be omitted in the rest): Given $a(i, j)$ and $E(i-1, j)$ let us denote by $W^*_{E(i-1,j)}(a(i, j))$ the element $\tilde{a}(i, j)$ of $\mathscr{C}_{i,j}$ such that $\tilde{a}_l\tilde{a}_{l+1} = a_{l+1}a_l$ if $E_{l-1}E_lE_{l+1} = \bar{\mathrm{e}}\mathrm{e}\bar{\mathrm{e}}$, $i \le l \le j-1$, and $\tilde{a}_l = a_l$ otherwise. In the summations below, ω^{ℓ} and $\omega^{\boldsymbol{r}}$ run over $\{\mathrm{e}, \bar{\mathrm{e}}\}^2$, η^{ℓ} and $\eta^{\boldsymbol{r}}$

over $\{0, 1\}^2$, and $\boldsymbol{a}''$ over $\mathscr{C}_{-N+2,N-2}$, respectively. Then

$$
\begin{aligned}
(3.9)\quad & S_N(\mathrm{ee}\omega_{-N+2}, \omega_{N-3}\bar{\mathrm{e}}\bar{\mathrm{e}}; \mathrm{ee}\omega_{-N+2}, \omega_{N-3}\bar{\mathrm{e}}\mathrm{e}) \\
&= \sum_{\boldsymbol{a}}\sum_{\boldsymbol{H}}\left|\sum_{\omega^\ell}\sum_{\omega^r}\sum_{\eta^\ell}\sum_{\eta^r}\left\{\frac{\check{\nu}(W_{\omega^\ell \mathrm{ee}\boldsymbol{E}''\bar{\mathrm{e}}\bar{\mathrm{e}}\omega^r}(\eta^\ell \boldsymbol{a}\eta^r))}{Q(\boldsymbol{a}^{(\ell)}, \mathrm{e})Q(\boldsymbol{a}^{(r)}, \bar{\mathrm{e}})} - \frac{\check{\nu}(W_{\omega^\ell \mathrm{ee}\boldsymbol{E}''\bar{\mathrm{e}}\mathrm{e}\omega^r}(\eta^\ell \boldsymbol{a}\eta^r))}{Q(\boldsymbol{a}^{(\ell)}, \mathrm{e})Q(\boldsymbol{a}^{(r)}, \mathrm{e})}\right\}\right| \\
&= \sum_{\boldsymbol{a}}\sum_{\boldsymbol{H}}\left|\sum_{\omega^r}\sum_{\eta^r}\left\{\frac{Q(\boldsymbol{a}^{(\ell)}W^*_{\mathrm{e}\boldsymbol{E}''\bar{\mathrm{e}}}(\boldsymbol{a}'')\xi^{r,\bar{\mathrm{e}}}, \mathrm{ee}\boldsymbol{E}''\bar{\mathrm{e}}\bar{\mathrm{e}}\omega^r)}{Q(\boldsymbol{a}^{(\ell)}, \mathrm{e})Q(\boldsymbol{a}^{(r)}, \bar{\mathrm{e}})}\nu(\boldsymbol{a}^{(\ell)}W^*_{\mathrm{e}\boldsymbol{E}''\bar{\mathrm{e}}}(\boldsymbol{a}'')\xi^{r,\bar{\mathrm{e}}})\right.\right. \\
&\qquad \left.\left. - \frac{Q(\boldsymbol{a}^{(\ell)}W^*_{\mathrm{e}\boldsymbol{E}''\bar{\mathrm{e}}}(\boldsymbol{a}'')\xi^{r,\mathrm{e}}, \mathrm{ee}\boldsymbol{E}''\bar{\mathrm{e}}\mathrm{e}\omega^r)}{Q(\boldsymbol{a}^{(\ell)}, \mathrm{e})Q(\boldsymbol{a}^{(r)}, \mathrm{e})}\nu(\boldsymbol{a}^{(\ell)}W^*_{\mathrm{e}\boldsymbol{E}''\bar{\mathrm{e}}}(\boldsymbol{a}'')\xi^{r,\mathrm{e}})\right\}\right|,
\end{aligned}
$$

where $\boldsymbol{E}'' = \omega_{-N+2}\boldsymbol{H}\omega_{N-3}$ and $\xi^{r,\bar{\mathrm{e}}[\mathrm{resp.}\,\mathrm{e}]} = W^*_{\bar{\mathrm{e}}\bar{\mathrm{e}}[\mathrm{resp.}\,\bar{\mathrm{e}}\mathrm{e}]\omega^r}(\boldsymbol{a}^{(r)}\eta_{N+1})\eta_{N+2}$. Since $W^*_{\mathrm{e}\boldsymbol{E}''\bar{\mathrm{e}}}(\mathscr{C}_{-N+2,N-2}) = \mathscr{C}_{-N+2,N-2}$, the above equals

$$
\begin{aligned}
&\sum_{\boldsymbol{a}^{(\ell)}}\sum_{\boldsymbol{a}^{(r)}}\sum_{\boldsymbol{H}}\sum_{\boldsymbol{a}''}\left|\sum_{\omega^r}\sum_{\eta^r}\left\{\frac{Q(\boldsymbol{a}^{(\ell)}\boldsymbol{a}''\xi^{r,\bar{\mathrm{e}}}, \mathrm{ee}\boldsymbol{E}''\bar{\mathrm{e}}\bar{\mathrm{e}}\omega^r)}{Q(\boldsymbol{a}^{(\ell)}, \mathrm{e})Q(\boldsymbol{a}^{(r)}, \bar{\mathrm{e}})}\nu(\boldsymbol{a}^{(\ell)}\boldsymbol{a}''\xi^{r,\bar{\mathrm{e}}})\right.\right. \\
&\qquad\qquad \left.\left. - \frac{Q(\boldsymbol{a}^{(\ell)}\boldsymbol{a}''\xi^{r,\mathrm{e}}, \mathrm{ee}\boldsymbol{E}''\bar{\mathrm{e}}\mathrm{e}\omega^r)}{Q(\boldsymbol{a}^{(\ell)}, \mathrm{e})Q(\boldsymbol{a}^{(r)}, \mathrm{e})}\nu(\boldsymbol{a}^{(\ell)}\boldsymbol{a}''\xi^{r,\mathrm{e}})\right\}\right|
\end{aligned}
$$

$$
\begin{aligned}
&= \sum_{\boldsymbol{a}}\left|\sum_{\omega^r}\sum_{\eta^r}\right. \\
&\qquad \left\{\frac{Q(a(-N, -N+3), \mathrm{ee}\omega_{-N+2})Q(a_{N-3}a_{N-2}\xi^{r,\bar{\mathrm{e}}}, \omega_{N-3}\bar{\mathrm{e}}\bar{\mathrm{e}}\omega^r)}{Q(\boldsymbol{a}^{(\ell)}, \mathrm{e})Q(\boldsymbol{a}^{(r)}, \bar{\mathrm{e}})}\nu(\boldsymbol{a}^{(\ell)}\boldsymbol{a}''\xi^{r,\bar{\mathrm{e}}})\right. \\
&\qquad \left.\left. - \frac{Q(a(-N, -N+3), \mathrm{ee}\omega_{-N+2})Q(a_{N-3}a_{N-2}\xi^{r,\mathrm{e}}, \omega_{N-3}\bar{\mathrm{e}}\mathrm{e}\omega^r)}{Q(\boldsymbol{a}^{(\ell)}, \mathrm{e})Q(\boldsymbol{a}^{(r)}, \mathrm{e})}\nu(\boldsymbol{a}^{(\ell)}\boldsymbol{a}''\xi^{r,\mathrm{e}})\right\}\right| \\
&\qquad\qquad\qquad \left[\text{by } \sum_{\boldsymbol{H}} Q(a(-N+3, N-3), \boldsymbol{H}) = 1\right] \\
&= \sum_{\boldsymbol{a}} Q(a(-N+1, -N+3), \mathrm{e}\omega_{-N+2}) \\
&\qquad \times \left|\frac{Q(a(N-3, N-1), \omega_{N-3}\bar{\mathrm{e}})}{Q(\boldsymbol{a}^{(r)}, \bar{\mathrm{e}})}\sum_{\eta^r}\Delta^{N,N+1}(\boldsymbol{a}\eta^r)\right. \\
&\qquad\qquad \left. - \frac{Q(a(N-3, N-2), \omega_{N-3})}{Q(\boldsymbol{a}^{(r)}, \mathrm{e})}\sum_{\eta_{N+1}}\Delta^{N-1,N}(\boldsymbol{a}\eta_{N+1})\right|
\end{aligned}
$$

by $\xi^{r,\bar{\mathrm{e}}} = a_{N-1}\eta_{N+1}a_N\eta_{N+2}$ if $\omega^r = \mathrm{e}\bar{\mathrm{e}}$ and so on. Here $\Delta^{I,I+1}(\boldsymbol{b})$, $\boldsymbol{b}\in\mathscr{C}_{i,j}$, $i < I < j-1$, is given by

$$\Delta^{I,I+1}(\boldsymbol{b}) = Q(b(I-1, I+2)^{I,I+1}, \bar{\mathrm{e}}\mathrm{e}\bar{\mathrm{e}})\nu(\boldsymbol{b}^{I,I+1}) - Q(b(I-1, I+2), \bar{\mathrm{e}}\mathrm{e}\bar{\mathrm{e}})\nu(\boldsymbol{b})$$

and $\boldsymbol{b}^{I,I+1} = {}_i[b_i \cdots b_{I-1}b_{I+1}b_Ib_{I+2}\cdots b_j]_j$. Similarly we have

$$\begin{aligned}(3.10)\qquad & S_N(\mathrm{ee}\omega_{-N+2}, \mathrm{ee\bar{e}}; \mathrm{ee}\omega_{-N+2}, \mathrm{eee}) \\ & = \sum_a Q(a(-N+1, -N+3), \mathrm{e}\omega_{-N+2}) \\ & \qquad \times \left| \frac{Q(a(N-3, N-1), \mathrm{ee})}{Q(a^{(r)}, \bar{\mathrm{e}})} \sum_{\eta^r} \Delta^{N,N+1}(a\eta^r) \right|.\end{aligned}$$

Then by (3.8) and (3.10)

$$\lim_{N\to\infty} \sum_a \left| \sum_{\eta^r} \Delta^{N,N+1}(a\eta^r) \right| = 0, \tag{3.11}$$

which together with (3.9) yields

$$\lim_{N\to\infty} \sum_a \left| \sum_{\eta_{N+1}} \Delta^{N-1,N}(a\eta_{N+1}) \right| = 0.$$

On the other hand we have

$$\begin{aligned} D_N(\mathrm{ee}, \omega_{N-2}\bar{\mathrm{e}}) &= \sum_a Q(a(-N, -N+2), \mathrm{ee}) Q(a_{N-2}a_{N-1}, \omega_{N-2}) \\ & \qquad \times \left| \sum_{\eta^r} \Delta^{N,N+1}(a\eta^r) \right|, \\ D_N(\mathrm{ee}, \bar{\mathrm{e}}\mathrm{e}) &= \sum_a Q(a(-N, -N+2), \mathrm{ee}) \left| \sum_{\eta_{N+1}} \Delta^{N-1,N}(a\eta_{N+1}) \right|, \end{aligned}$$

which gives us $\lim_{N\to\infty} D_N(\mathrm{ee}, \omega_{N-2}\bar{\mathrm{e}}) = \lim_{N\to\infty} D_N(\mathrm{ee}, \bar{\mathrm{e}}\mathrm{e}) = 0$.

To treat $D_N(\varepsilon^\ell, \varepsilon^r)$, $\varepsilon^\ell \neq \mathrm{ee}$, let us put

$$\begin{aligned}\Delta^{I,I+1}_{J,J+1}(b) &= Q(b(I-1, I+2)^{I,I+1}, \bar{\mathrm{e}}\mathrm{e}\bar{\mathrm{e}}) \Delta^{J,J+1}(b^{I,I+1}) \\ & \qquad - Q(b(I-1, I+2), \bar{\mathrm{e}}\mathrm{e}\bar{\mathrm{e}}) \Delta^{J,J+1}(b).\end{aligned}$$

By combining

$$\begin{aligned} & S_N(\bar{\mathrm{e}}\omega_{-N+1}\omega_{-N+2}, \mathrm{ee\bar{e}}; \bar{\mathrm{e}}\omega_{-N+1}\omega_{-N+2}, \mathrm{eee}) \\ & = \sum_a \frac{Q(a(-N+1, -N+3), \omega_{-N+1}\omega_{-N+2})}{Q(a^{(\ell)}, \bar{\mathrm{e}})} \cdot \frac{Q(a(N-3, N-1), \mathrm{ee})}{Q(a^{(r)}, \bar{\mathrm{e}})} \\ & \qquad \times \left| \sum_{\eta^r} \Delta^{N,+1}(a\eta^r) + \sum_{\eta\ell\eta^r} \Delta^{N,N+1}_{-N-1,-N}(\eta^\ell a\eta^r) \right| \end{aligned}$$

with (3.11) we have

$$\lim_{N\to\infty} \sum_a \left| \sum_{\eta^\ell} \sum_{\eta^r} \Delta^{N,N+1}_{-N-1,-N}(\eta^\ell a\eta^r) \right| = 0.$$

Just as (3.11) we also have

$$\lim_{N\to\infty} \sum_a \left| \sum_{\eta^\ell} \Delta^{-N-1,-N}(\eta^\ell a) \right| = 0.$$

These results imply $\lim_{N\to\infty} D_N(\bar{\mathrm{e}}\omega_{-N+1}, \omega_{N-2}\bar{\mathrm{e}}) = 0$. Indeed we have only to use the relation

$$D_N(\bar{\mathrm{e}}\omega_{-N+1}, \omega_{N-2}\bar{\mathrm{e}}) = \sum_a Q(a_{-N+1}a_{-N+2}, \omega_{-N+1}) Q(a_{N-2}a_{N-1}, \omega_{N-2})$$

$$\times\,|Q(a_{N-1}a_N,\bar{e})\textstyle\sum_{\eta^\ell}\Delta^{-N-1,-N}(\eta^\ell a)+Q(a_{-N}a_{-N+1},\bar{e})\sum_{\eta^r}\Delta^{N,N+1}(a\eta^r)$$
$$+\textstyle\sum_{\eta^\ell}\sum_{\eta^r}\Delta^{N,N+1}_{-N-1,-N}(\eta^\ell a\eta^r)|.$$

The remaining D_N's can be treated analogously.

4°. Now we can complete the proof as follows: It is obvious from (3.6) and (3.7) that $\lim_{N\to\infty}F_N^{(2)}(v)=0$ holds. The equality $\lim_{N\to\infty}F_N^{(1)}(v)=0$ is obtained immediately from (3.7) and the fact that for every $\varepsilon>0$ there exists $M_\varepsilon>0$ such that $|u\log u-v\log v|<\varepsilon+M_\varepsilon|u-v|$ for every $0\leq u,v\leq 1$. Thus we have $\lim_{N\to\infty}F_N(v)=0$. This with (2.4) and the monotonicity of $F_N(v)$, which follows from Lemma 1, yield $F_N(v)=0$, $N\in\mathbf{N}$. □

Concluding remarks. 1. If $\beta=0$, our argument for the proof of necessity part of Theorem 2 does not go through because, for example, $\lim_{N\to\infty}(3.10)=0$ does not immediately imply (3.11) (there occurs a case that $Q(\zeta,E)=0$). To treat this case a more precise argument will be needed.

2. Our argument also does not go through in the asymmetric case such as $Q(01,\mathrm{e})=\alpha$, $Q(10,\mathrm{e})=0$ and $Q(11,\bar{\mathrm{e}})=Q(00,\bar{\mathrm{e}})=1$, because (2.1) is not equivalent to $(\mathrm{RCP})_\gamma$. This case was treated in [9], and it was shown there that for $0<\alpha\leq 1/2$ a (nontrivial) probability measure on $\mathscr{X}$ is stationary for the above asymmetric exclusion process iff it has $(\mathrm{RCP})_{1-\alpha}$. The proof was based on the method of coupled Markov process.

3. We can consider another limiting procedure different from Theorem 1. Let $P(\eta,A)$ be the transition probabilities which define $(\mathrm{EP})_{\alpha,\beta}$. For fixed $n\in\mathbf{N}$ define new transition probabilities $P^*(\eta,A)_n$ by

$$P^*(\eta,{}_i[a_i\cdots a_j]_j)_n=\begin{cases}\dfrac{1}{n}P(\eta,{}_i[a_i\cdots a_j]_j) & \text{if } a_i\cdots a_j\neq\eta_i\cdots\eta_j\\[2ex] 1-\dfrac{1}{n}+\dfrac{1}{n}P(\eta,{}_i[a_i\cdots a_j]_j) & \text{otherwise},\end{cases}$$

and denote by $S_n^*(k)$, $k=0,1,\cdots$, the corresponding semigroup. Let $\tilde{T}^*(t)$ be the semigroup on $C(\mathscr{X})$ defined by the bounded operator

$$(A_\infty^*f)(\eta)=\int_{\mathscr{X}}P(\eta,d\xi)f(\xi)-f(\eta),\quad f\in C(\mathscr{X})$$

(we refer to Example 2.3 and Proposition 2.8 in §2, Chap. I of [7]). Then we have

$$\lim_{n\to\infty}\|S_n^*([tn])f-\tilde{T}^*(t)f\|_\infty=0 \text{ for all } t\geq 0$$

for $f\in C(\mathscr{X})$. We remark that in the Markov process associated with A_∞^* i.m.p.'s can change their position at the same time, and that a probability measure ν on $\mathscr{X}$ is stationary for A_{∞}^{*} if and only if it has $(\mathrm{RCP})_{\gamma}$. Indeed the condition that ν is stationary for A_{∞}^{*} is equivalent to the one that ν is stationary for $(\mathrm{EP})_{\alpha,\beta}$, that is, for $\boldsymbol{a}\in\mathscr{C}$

$$\int_{\mathscr{X}} A_{\infty}^{*}\chi_{\boldsymbol{a}}(\eta)d\nu(\eta)=0 \iff \int_{\mathscr{X}} d\nu(\eta)P(\eta, \boldsymbol{a})=\nu(\boldsymbol{a}).$$

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