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MULTIPLE QUANTIFICATION AND THE USE OF SPECIAL QUANTIFIERS IN EARLY SIXTEENTH CENTURY LOGIC

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I have three reasons for writing this paper. In the first place, I want to explain the early sixteenth century practice of using the letters 'a', 'b', 'c', and 'd' as special signs governing the interpretation of terms within sentences.[1] In the second place, I want to investigate the analysis which logicians in the medieval tradition gave of such sentences as "There is somebody all of whose donkeys are running", "Everybody has at least one donkey which is running", and "At least one of the donkeys which everybody owns is running".[2] In the third place, I want to show that, despite what Geach has suggested,[3] logicians in the medieval tradition were capable of offering good reasons for rejecting such inferences as "Every boy loves some girl, therefore there is some girl that every boy loves". My discussion will be based mainly on the work of a group of logicians who were at the University of Paris in the first two decades of the sixteenth century, in particular Fernando de Enzinas, Antonio Coronel, and Domingo de Soto.

In order to make sense of the non-standard cases I shall be investigating, it is first of all necessary to describe how logicians of the period in question analyzed standard propositions of the form "Every *A* is *B*", "No *A* is *B*", "Some *A* is *B*", and "Some *A* is not *B*". Their main tool was the doctrine of personal supposition; and there were said to be four kinds of personal supposition, distributive, collective, determinate, and merely confused. If a term had distributive supposition, then the sentence in which it appeared was said to be equivalent to a conjunction of sentences, each of which contained a different singular term in place of the term with distributive supposition. Thus, "Every donkey is running" is equivalent to "Donkey$_1$ is running and donkey$_2$ is running and . . . and donkey$_n$ is running". If a term had collective supposition, then the sentence in which it appeared was said to be equivalent to a sentence in which the term with collective supposition was replaced by a conjunction of singular terms. Thus "All the apostles are twelve" is equivalent to "Apostle$_1$ and apostle$_2$ and . . . and apostle$_n$ are twelve". Collective supposition was little used. If a term had determinate supposition, then the sentence in which it

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appeared was said to be equivalent to a disjunction of sentences, each of which contained a different singular term in place of the term with determinate supposition. Thus, "Some donkey is running" is equivalent to "Donkey_1 is running or donkey_2 is running or . . . or donkey_n is running". If a term had merely confused supposition, then the sentence in which it appeared was said to be equivalent to a sentence in which the term with merely confused supposition was replaced by a disjunction of singular terms. Thus, "Every donkey is running" is equivalent to "Every donkey is running thing_1 or running thing_2 or . . . or running thing_n".

In order to make full use of the doctrine of personal supposition, two kinds of rule had to be offered, one of which determined the type of supposition the terms in a sentence had, and one of which determined the way in which the sentence as a whole was to be analyzed. I will mention only a few of the rules of the first kind. A term governed directly by 'all' or any other universal sign had distributive supposition. A term governed indirectly by such signs had merely confused supposition. A term governed by 'some' or any other particular sign had determinate supposition. A term governed by 'not' had distributive supposition. Thus, in "Every *A* is *B*" *A* has distributive supposition and *B*, which is governed indirectly by 'all', has merely confused supposition. "No *A* is *B*" was thought of as equivalent to "Every *A* is not *B*", so that both *A* and *B* have distributive supposition. In "Some *A* is *B*", both *A* and *B* have determinate supposition. In "Some *A* is not *B*", *A* has determinate supposition and *B* has distributive supposition.

The second kind of rule determined the way in which a sentence as a whole was to be analyzed by determining the order in which quantified terms were to be replaced by singular terms.[4] It should be noted that this process was called 'descent', and that the reverse process was called 'ascent'.[5] The rule for descent was: first, replace terms with determinate supposition; second, replace terms with distributive supposition; third, replace terms with merely confused supposition. Collective supposition was not mentioned. In cases where both the terms in a proposition of standard form had either determinate or distributive supposition, no order of priority was given. Since both 'and' and 'or' are commutative, one will get logically equivalent results whichever term one begins with.

Further rules were given to determine the type of supposition which the terms in a proposition's contradictory would have.[6] If the proposition to be contradicted contained one term with distributive supposition and one term with merely confused supposition, then its contradictory would contain one term with determinate supposition and one term with distributive supposition. If the proposition to be contradicted contained one term with distributive supposition and one term with determinate supposition, then its contradictory would contain one term with merely confused supposition and one term with distributive supposition. Thus, "Every *A* is *B*" and "Some *A* is not *B*" are contradictories. If the proposition to be contradicted contained two terms with distributive supposition, then its contradictory would contain two terms with determinate supposition. If it contained two terms with determinate supposition, then its contradictory would contain two terms with distributive supposition. Thus, "No A is B" and "Some A is B" are contradictories.

The practical results of all these rules can best be understood by studying the following analyses of the four standard forms of proposition:

Every A is B:

$$(A_1 = B_1 \vee B_2 \vee \ldots \vee B_n) \,.\, (A_2 = B_1 \vee B_2 \vee \ldots \vee B_n) \,.\,.\,.\,. \\ .\, (A_n = B_1 \vee B_2 \vee \ldots \vee B_n)$$

No A is B:

$$(A_1 \neq B_1 \,.\, A_1 \neq B_2 \,.\,.\,.\,. \,.A_1 \neq B_n) \,.\, (A_2 \neq B_1 \,.\, A_2 \neq B_2 \,.\,.\,.\,.\,.\, A_2 \neq B_n) \\ .\,.\,.\,.\,.\, (A_n \neq B_1 \,.\, A_n \neq B_2 \,.\,.\,.\,.\,.\, A_n \neq B_n)$$

Some A is B:

$$(A_1 = B_1 \vee A_1 = B_2 \vee \ldots \vee A_1 = B_n) \vee (A_2 = B_1 \vee A_2 = B_2 \vee \ldots \vee \\ A_2 = B_n) \vee \ldots \vee (A_n = B_1 \vee A_n = B_2 \vee \ldots \vee A_n = B_n)$$

Some A is not B:

$$(A_1 \neq B_1 \,.\, A_1 \neq B_2 \,.\,.\,.\,.\,.\, A_1 \neq B_n) \vee (A_2 \neq B_1 \,.\, A_2 \neq B_2 \,.\,.\,.\,.\,. \\ A_2 \neq B_n) \vee \ldots \vee (A_n \neq B_1 \,.\, A_n \neq B_2 \,.\,.\,.\,.\,.\, A_n \neq B_n)$$

In subsequent discussion I shall continue to use the same symbolism, but in order to simplify matters I shall make the arbitrary assumption that each class contains only three members.

To any logician who reflects upon the matter, it is bound to be evident that not all sentences of an apparently standard form need have a standard interpretation, and that not all sentences are of a standard form, in the sense that not all fit into the pattern of A, E, I, and O propositions. There was considerable discussion of sentences of non-standard form in medieval logicians, but no new techniques were invented. It was only towards the end of the fifteenth century that logicians began to use first 'a' and 'b', and then the subsequent letters of the alphabet, as special signs both to alter the interpretation of sentences of standard form, and to make clear the interpretation of and relationships between sentences of non-standard form.[7]

The two most commonly used signs were 'a' and 'b'. 'a' was used to indicate that the term following it had merely confused supposition; and a favorite example was "a. man is not an animal" [*a. homo non est animal*],[8] which, given the extensionalist analysis of terms is true, because it is equivalent to:

$$((A_1 \vee A_2 \vee A_3) \neq B_1) \vee ((A_1 \vee A_2 \vee A_3) \neq B_2) \vee ((A_1 \vee A_2 \vee A_3) \neq B_3)$$

That is, if each of several men $[A_i]$ is identical to a different animal $[B_i]$ it is true to say of each animal that one or more men is not identical to that animal. One can even say truly that "a. man is not a man".[9] However, if the term used has only one referent, as in "a. phoenix is not a phoenix", the sentence will be false.[10] 'b' was used to indicate that the term following it had determinate supposition; and a favourite example was "Every man is b. animal" [*Omnis homo est b. animal*].[11] This is false because it is equivalent to:

$$(A_1 = B_1 \,.\, A_2 = B_1 \,.\, A_3 = B_1) \vee (A_1 = B_2 \,.\, A_2 = B_2 \,.\, A_3 = B_2) \\ \vee (A_1 = B_3 \,.\, A_2 = B_3 \,.\, A_3 = B_3)$$

That is, every man $[A_i]$ is identical to the very same animal $[B_i]$. One might think that 'some' could have been used instead of 'b' in this context, but it was felt that in "Every man is some animal" [*Omnis homo est aliquod animal*] the rule that 'every' produces merely confused supposition in the terms it governs indirectly overrode the rule that 'some' produces determinate supposition.[12] 'b' was a sign which could not be overridden by any other sign.

How propositions stood in relation to the standard table of opposition was a topic of great interest to logicians in the medieval tradition, and obviously the use of 'a' and 'b' made some further rules necessary. Domingo de Soto gave two.[13] The first was that a pair of propositions which would be contradictory if one of the distributed terms were related to a term with merely confused supposition, rather than to a term with determinate supposition, were contraries. Thus "Every A is b.B" and "Some A is not B" are contraries. They cannot both be true, but they can both be false. If it is false that some man is not disputing, it will be true that every man is disputing, but it certainly need not be true that every man is the same disputant. To argue that it was true, said Domingo de Soto, would be to violate the standard suppositional rule that a sentence containing both a distributive term and a term with determinate supposition cannot be derived from a sentence in which the same terms have respectively distributive and merely confused supposition. That is, one cannot argue "Every A is B, therefore every A is b.B". The second rule was that a pair of propositions which would be contradictory if one of the disputed terms were related to a term with determinate supposition, rather than to a term with merely confused supposition, are subcontraries. Thus, "Every A is B" and "a.A is not B" are subcontraries. They may both be true, but they cannot both be false. If it is false that all men are disputing, then it will be true that at least one of the men is not identical to a given disputant. Again, if it is false that at least one of the men is not identical to a given disputant, it will follow that they are all identical to some disputant or other. Domingo de Soto could have gone further, and pointed out that the two propositions containing the new signs, "Every A is b. B" and "a. A is not B" were contradictory to each other by the standard rules mentioned above. One would thus obtain the following square of opposition:

Every A is b.B	contraries	Some A is not B
subalterns	contradictories contradictories	subalterns
Every A is B	subcontraries	a.A is not B

'a' and 'b' were sometimes employed to make possible the simple conversion of certain kinds of proposition.[14] "No *A* is *B*" and "Some *A* is *B*" can both be simply converted in the sense that subject and predicate can be interchanged without the sentence undergoing any change in truth-value or indeed any significant change in analysis, given that both the terms in each sentence have the same type of supposition. This is not true of "Every *A* is *B*" and "Some *A* is not *B*", where changing the order of subject and predicate will also change the type of supposition that each term possesses. However, if "Every *A* is *B*" [*Omnis homo est animal*] is converted to "a.*B* is every *A*" [*a.animal omnis homo est*] and "Some *A* is not *B*" [*homo non est lapis*] to "Every *B* is not b.*A*" [*Omnis lapis b.homo non est*], the terms retain their supposition and the converted sentences are equivalent in truth-value to the original sentences.

In some cases a sentence containing 'a' or 'b' was said to be equivalent to a sentence similar except that it did not contain such a special sign. Most of the cases cited involved sentences with a non-standard form, but Domingo de Soto gave one instance of sentences which had only two terms and which did not contain any term with distributive supposition.[15] In "Some *A* is *B*" and "a.*A* is *B*", he said, the term with determinate supposition is equivalent to the term with confused supposition. His reasons become obvious if one considers the following analyses:

Some *A* is *B*:

$$(A_1 = B_1 \vee A_2 = B_1 \vee A_3 = B_1) \vee (A_1 = B_2 \vee A_2 = B_2 \vee A_3 = B_2)$$
$$\vee (A_1 = B_3 \vee A_2 = B_3 \vee A_3 = B_3)$$

a.*A* is *B*:

$$((A_1 \vee A_2 \vee A_3) = B_1) \vee ((A_1 \vee A_2 \vee A_3) = B_2) \vee ((A_1 \vee A_2 \vee A_3) = B_3)$$

Propositions of standard form, whether containing extra signs or not, raised few problems for the logician, but propositions of non-standard form are considerably more difficult to deal with, especially when they are to be analyzed by means of singular terms and the copula of identity. I shall examine two kinds of non-standard proposition, each containing three terms. One kind, which I shall mention briefly at the end of this paper, contains a noun in the nominative case, a verb, and a direct object; the other kind contains a noun in the nominative case, a noun in the genitive case, and a verb. I am thinking here of such sentences as "Every man's donkey is running". If one takes a sentence like the one just mentioned, which contains a universal sign, a nominative and a genitive, then there are two main possibilities to be considered: either the universal sign governs the noun in the nominative case, or it governs the noun in the genitive case. An example of the first is: "Every donkey of some man is running" [*Quilibet asinus hominis currit*]; an example of the second is: "Of every man some donkey is running" [*Cuiuslibet hominis asinus currit*].[16] Each main possibility gives rise to two further possibilities. In the first case, the universal sign '*quilibet*' either precedes the genitive or it does not, as in "Of some man every donkey is running" [*Hominis quilibet asinus currit*]. In the second case, the universal sign '*cuiuslibet*' either precedes the nominative or it does not, as in "Some donkey of every man is running" [*Asinus cuiuslibet hominis currit*]. Yet further possibilities arise when one realizes that the remaining terms may themselves be governed by universal or particular signs. The following table will serve as a guide to my discussion. I have added the cases in which no universal signs appeared, though these were not normally the focus of attention.

Let A be 'donkey', B be 'man' and C be 'running'.

I. *Universal Sign governs Noun in Nominative Case*

1. *Universal Sign precedes Noun in Genitive Case*

a. Every A of B is C	*Quilibet asinus hominis currit.*
[b. Some A of B is C	*Asinus hominis currit*]

2. *Universal Sign follows Noun in Genitive Case*

a. Of some B every A is C	*Hominis quilibet asinus currit.*
b. Of every B every A is C	*Cuiuslibet hominis quilibet asinus currit*
[c. Of some B some A is C	*Hominis asinus currit.*]

II. *Universal Sign governs Noun in Genitive Case*

1. *Universal Sign precedes Noun in Nominative Case*

a. Of every B every A is C	*Cf.* I2b.
b. Of every B some A is C	*Cuiuslibet hominis asinus currit.*
[c. Of some B some A is C	*Cf.* I2c.]

2. *Universal Sign follows Noun in Nominative Case*

a. Every A of every B is C	*Quilibet asinus cuiuslibet hominis currit.*
b. Some A of every B is C	*Asinus cuiuslibet hominis currit.*
[c. Some A of some B is C	*Cf.* I1b.]

The first kind of sentence, exemplified by "Every donkey of man is running", is the easiest to analyze since it was agreed that 'donkey of man' [*asinus hominis*] should be treated as a single term.[17] In this context it has distributive supposition, and had there been a particular sign instead of a universal sign, it would have had determinate supposition.[18] Thus, although the sentence apparently contains three terms, it can be treated as having a standard A form. A few logicians did discuss the status of '*hominis*' separately,[19] but their discussion did nothing to alter the overall analysis of the sentence. Some suggested that '*hominis*' had merely confused supposition, although the aggregate was distributed; while others suggested that it was distributed, but only *secundum quid*, since one could not descend from it. If one were to infer that every donkey of this man is running and every donkey of this man is running and so on, one would be faced with the possibility of a true antecedent and a false consequent in the case where not all men are donkey-owners.[20] The original sentence gives us no means of telling whether all men or only some are donkey-owners, or whether their ownership is collective or individual. Nor can we tell whether all donkeys have human owners or not. In the fourteenth century Albert of Saxony set out very clearly what one could and could not infer from the sentence in question.[21] One cannot argue "Every donkey of man is running, Brunellus is a donkey, therefore Brunellus is running", because Brunellus might be a wild donkey; nor can one argue "Every donkey of man is running, Socrates is a man, therefore a donkey of Socrates is running", for Socrates might not own any donkeys. However, one can argue: "Every donkey of man is running, Brunellus is a donkey of man, therefore Brunellus is running".

In the discussion of the second type of sentence, exemplified by "Of some man every donkey is running" (I2a) it is made quite clear that 'of man' and 'donkey' are to be treated separately. 'Donkey' continues to have distributive supposition, and 'running' to have merely confused supposition, since it is indirectly governed by the universal sign 'every', but 'of man' now has determinate supposition, since it is taken to be governed by a particular sign.[22] It could also have distributive supposition, if it were governed by '*cuiuslibet*', the genitive form of '*quilibet*'. A special rule was given to determine the order of descent, namely that one should always descend from the modifier [*determinatio*] before the modifiable [*determinabile*].[23] That is, 'man' takes precedence over 'donkey', even in the case where 'man' does not have determinate supposition. This rule was introduced to prevent invalid inferences. For instance, from "Of every man every donkey is running" one cannot validly infer "Of every man this donkey is running" because if each man owns one donkey, and all the donkeys are running, then the antecedent is true, but the consequent, which implies collective ownership, is false.[24]

The instructions for analyzing the two propositions in question (I2a and I2b) seem clear enough at first sight. In the first proposition B has determinate supposition and we should therefore begin by replacing the original sentence with a disjunction of sentences each of which contains the singular term B_i (i = 1 to n). A has distributive supposition, so we are to analyze each disjunct into a conjunction of sentences each of which contains the singular term A_i (i = 1 to n). C has merely confused supposition, so it is to be replaced by the disjunction of C_1, C_2 and so on up to C_n. Given my arbitrary assumption that there are three members of each class, the result will look like this:

$$[(\text{Of } B_1A_1 = C_1 \vee C_2 \vee C_3) \,.\, (\text{Of } B_1A_2 = C_1 \vee C_2 \vee C_3) \,.\, (\text{Of } B_1A_3 = C_1 \vee C_2 \vee C_3)] \vee$$
$$[(\text{Of } B_2A_1 = C_1 \vee C_2 \vee C_3) \,.\, (\text{Of } B_2A_2 = C_1 \vee C_2 \vee C_3) \,.\, (\text{Of } B_2A_3 = C_1 \vee C_2 \vee C_3)] \vee$$
$$[(\text{Of } B_3A_1 = C_1 \vee C_2 \vee C_3) \,.\, (\text{Of } B_3A_2 = C_1 \vee C_2 \vee C_3) \,.\, (\text{Of } B_3A_3 = C_1 \vee C_2 \vee C_3)]$$

The second proposition is like the first except that B has distributive supposition. Since B is the modifier, we begin with B and replace the original sentence with a conjunction of sentences each of which contains the singular term B_i (i = 1 to n). We then proceed as above. The result will look like this:

$$[(\text{Of } B_1A_1 = C_1 \vee C_2 \vee C_3) \,.\, (\text{Of } B_1A_2 = C_1 \vee C_2 \vee C_3) \,.\, (\text{Of } B_1A_3 = C_1 \vee C_2 \vee C_3)] \,.$$
$$[(\text{Of } B_2A_1 = C_1 \vee C_2 \vee C_3) \,.\, (\text{Of } B_2A_2 = C_1 \vee C_2 \vee C_3) \,.\, (\text{Of } B_2A_3 = C_1 \vee C_2 \vee C_3)] \,.$$
$$[(\text{Of } B_3A_1 = C_1 \vee C_2 \vee C_3) \,.\, (\text{Of } B_3A_2 = C_1 \vee C_2 \vee C_3) \,.\, (\text{Of } B_3A_3 = C_1 \vee C_2 \vee C_3)]$$

However, we are now faced with a problem of interpretation. It looks very much as if we are asserting that the very same donkeys are owned by each of the men, yet collective ownership has been denied, at least in the second case. Moreover, the analysis offered by Coronel, who included the case of a sentence with no universal signs, makes it clear that each man owns different donkeys. He wrote[25]:

[I2a] Of Socrates Brunellus is running and of Socrates Favellus is running and so on for each of his donkeys, or of Plato Grivellus is running and of Plato Grisellus is running and so on, therefore of some man every donkey is running.

[I2b] Of Socrates Brunellus is running and of Socrates Favellus is running and so on for each of his donkeys, and of Plato Grivellus is running and of Plato Grisellus is running and so on, therefore of every man every donkey is running.

[I2c] Either of Socrates Brunellus is running or of Socrates Favellus is running and so on for each of his donkeys, or of Plato Grivellus is running or of Plato Grisellus is running and so on, therefore of some man some donkey is running.

In the light of these examples it is obvious that instead of using A_i for all donkeys, no matter who they belong to, we should use D_i for the donkeys of the first man, E_i for the donkeys of the second man and F_i for the donkeys of the third man. We thus obtain:

I2a:

$$[(\text{Of } B_1D_1 = C_1 \vee C_2 \vee C_3) \,.\, (\text{Of } B_1D_2 = C_1 \vee C_2 \vee C_3) \,.\, (\text{Of } B_1D_3 = C_1 \vee C_2 \vee C_3)] \vee$$
$$[(\text{Of } B_2E_1 = C_1 \vee C_2 \vee C_3) \,.\, (\text{Of } B_2E_2 = C_1 \vee C_2 \vee C_3) \,.\, (\text{Of } B_2E_3 = C_1 \vee C_2 \vee C_3)] \vee$$
$$[(\text{Of } B_3F_1 = C_1 \vee C_2 \vee C_3) \,.\, (\text{Of } B_3F_2 = C_1 \vee C_2 \vee C_3) \,.\, (\text{Of } B_3F_3 = C_1 \vee C_2 \vee C_3)]$$

I2b:

$$[(\text{Of } B_1D_1 = C_1 \vee C_2 \vee C_3) \,.\, (\text{Of } B_1D_2 = C_1 \vee C_2 \vee C_3) \,.\, (\text{Of } B_1D_3 = C_1 \vee C_2 \vee C_3)] \,.$$
$$[(\text{Of } B_2E_1 = C_1 \vee C_2 \vee C_3) \,.\, (\text{Of } B_2E_2 = C_1 \vee C_2 \vee C_3) \,.\, (\text{Of } B_2E_3 = C_1 \vee C_2 \vee C_3)] \,.$$
$$[(\text{Of } B_3F_1 = C_1 \vee C_2 \vee C_3) \,.\, (\text{Of } B_3F_2 = C_1 \vee C_2 \vee C_3) \,.\, (\text{Of } B_3F_3 = C_1 \vee C_2 \vee C_3)]$$

I2c:

$$[(\text{Of } B_1D_1 = C_1 \vee C_2 \vee C_3) \vee (\text{Of } B_1D_2 = C_1 \vee C_2 \vee C_3) \vee (\text{Of } B_1D_3 = C_1 \vee C_2 \vee C_3)] \vee$$
$$[(\text{Of } B_2E_1 = C_1 \vee C_2 \vee C_3) \vee (\text{Of } B_2E_2 = C_1 \vee C_2 \vee C_3) \vee (\text{Of } B_2E_3 = C_1 \vee C_2 \vee C_3)] \vee$$
$$[(\text{Of } B_3F_1 = C_1 \vee C_2 \vee C_3) \vee (\text{Of } B_3F_2 = C_1 \vee C_2 \vee C_3) \vee (\text{Of } B_3F_3 = C_1 \vee C_2 \vee C_3)]$$

Thus, if we overlook the limited number of running things referred to, we have obtained a formal expression of these claims: "There is a man at least one of whose donkeys is running", "There is a man all of whose donkeys are running", and "For any man you take, all of that man's donkeys are running".

One example of the third kind of sentence (II1), in which the universal sign governs the noun in the genitive case and precedes the noun in the nominative case, has already been dealt with as I2b. This leaves us with "Of every man some donkey is running" (II1b) in which 'man' has distributive supposition, but both 'donkey' and 'running' are said to have merely confused supposition, because they are governed indirectly by the universal sign '*cuiuslibet*'.[26] Given the principles outlined above, this proposition can be analyzed as follows:

II1b:

$[(\text{Of } B_1 D_1 \vee D_2 \vee D_3) = (C_1 \vee C_2 \vee C_3)] \,.\, [(\text{Of } B_2 E_1 \vee E_2 \vee E_3) = (C_1 \vee C_2 \vee C_3)] \,.$
$[(\text{Of } B_3 F_1 \vee F_2 \vee F_3) = (C_1 \vee C_2 \vee C_3)]$

Domingo de Soto claimed that in the case where the resolution of a term with merely confused supposition was dependent on the resolution of a modifiable term, then that first term was equivalent to a term with determinate supposition, as in "Of every man some horse is running" [*Cuiuslibet hominis equus currit*] and "Of every man b.horse is running" [*Cuiuslibet hominis b. equus currit*].[27] This gives us the following analysis as an alternative:

II1b:

$[(\text{Of } B_1D_1 = C_1 \vee C_2 \vee C_3) \vee (\text{Of } B_1D_2 = C_1 \vee C_2 \vee C_3) \vee (\text{Of } B_1D_3 = C_1 \vee C_2 \vee C_3)] \,.$
$[(\text{Of } B_2E_1 = C_1 \vee C_2 \vee C_3) \vee (\text{Of } B_2E_2 = C_1 \vee C_2 \vee C_3) \vee (\text{Of } B_2E_3 = C_1 \vee C_2 \vee C_3)] \,.$
$[(\text{Of } B_3F_1 = C_1 \vee C_2 \vee C_3) \vee (\text{Of } B_3F_2 = C_1 \vee C_2 \vee C_3) \vee (\text{Of } B_3F_3 = C_1 \vee C_2 \vee C_3)]$

De Soto's claim makes good sense, since the sentence he offered as equivalent to the original sentence has the same pattern of analysis as the previous sentences I examined; and it also fits the pattern of analysis given by Coronel, which was:

[II1b] Of Socrates Brunellus is running or of Socrates Favellus is running and so on for each of his donkeys and of Plato Grivellus is running or of Plato Grisellus is running and so on, therefore of every man some donkey is running.

Thus, we have obtained a formal expression of the claim "For any man you take, at least one of that man's donkeys is running".

There was some confusion in earlier logicians as to what inferences could be made from II1b.[28] Albert of Saxony wrote that one could argue "Of every man some donkey is running, Brunellus is of a man, therefore Brunellus is running", and Dorp in his commentary on Buridan allowed "Of every man some donkey is running and Socrates is a man, therefore of Socrates some donkey is running", while the author of *Commentum* denied this very inference. It seems that Dorp is right, whereas both Albert of Saxony and the author of *Commentum* are wrong.

The fourth type of sentence (II2) remains to be dealt with. This type differs from the others in that it implies joint ownership of donkeys,[29] and hence there is no need to differentiate the donkeys belonging to one man from the donkeys belonging to another man. In II2 a both 'donkey' and 'man' have distributive supposition, and 'running' has merely confused supposition, but in II2b both 'donkey' and 'running' have determinate supposition. 'Man' retains its distributive supposition since it, and it alone, is governed by '*cuiuslibet*'.[30] The rule about modifiers and modifiables lays it down that in each case one should descend from 'man' before one descends from 'donkey', but Tartaretus pointed out that descending from 'man' at all could lead to invalid inferences.[31] If one argues "Every donkey of every man is running, therefore every donkey of this man is running" the antecedent will be true if there are three men who collectively own three donkeys, all of which are running, but the consequent will be false if each man owns in addition one personal donkey, which is not running. Moreover, if one argues "Some donkey of every man is running, therefore some donkey of this man is running" one moves from a sentence which is about collectively-owned donkeys to a sentence which is about individually-owned donkeys. Such a shift in reference is illegitimate. Tartaretus concluded that one cannot always descend from terms which have distributive supposition. It seems that the fourth type of sentence is more closely analogous to the first type of sentence (I1) than it is to the other two (I2 and II1). We are told something about donkeys in relation to their owners, but we do not know whether all or only some donkeys are collectively owned.

All four types of sentence could be altered further by the addition of 'a' or 'b', either to the predicate or to the noun in the subject-clause which was not governed by a universal quantifier. However, I shall not examine these possible variations directly. Instead I shall look at what was written about the contradictories of non-standard propositions. This topic was obviously one of importance to some early sixteenth century logicians, and a great many pages of extremely subtle and difficult discussion were devoted to it.[32] The main problem was this: given a proposition which contains terms with distributive, merely confused, and determinate supposition, what kind of supposition should the term with distributive supposition have in the proposition which is contradictory to the original proposition? According to Coronel, the answer was clear in two out of three cases.[33] First, if the term with determinate supposition is by itself the subject or predicate of the first proposition, then the term with distributive supposition will have merely confused supposition in the contradictory. Thus, the contradictory of "Of every man some donkey is b.donkey" [*Cuiuslibet hominis asinus est b.asinus*] is "Of a.man no donkey is a donkey" [*a.hominis nullus asinus est asinus*].[34] Second, if the term with merely confused supposition is by itself the subject or predicate of the first proposition, then the term with distributive supposition will have determinate supposition in the contradictory. Thus, the contradictory of "Of some man every donkey is a donkey" [*Hominis quilibet asinus est asinus*] is "Of every man b. donkey is not a donkey" [*Cuiuslibet hominis b. asinus non est asinus*].[35] The problem arose in the third case, where the distributive term is by itself the subject or predicate of the first proposition. Three solutions were mentioned by Coronel. One was that the aggregate of nominative and genitive should be assigned supposition as a whole, so that the standard rules could be used; and one was that the term with distributive supposition should have determinate supposition in the contradictory, but that an explicit reference to identity should be added. Thus, "Of some man a.donkey is not a donkey" [*Hominis a.asinus non est asinus*] would be contradicted by "Of every same man every donkey is b.donkey" [*Cuiuslibet eiusdem hominis quilibet asinus est b.asinus*]. The third, and most common solution, involved the use of further special signs, namely 'c' and 'd'.

'c' and 'd' were explained as follows: they make a term have mixed supposition, so that in a way it has both merely confused and determinate supposition. Coronel said that this meant that a term would be treated one way in descent and another way in ascent, but Enzinas and Domingo de Soto gave a more plausible account.[36] Enzinas took the sentence "Of some man a.donkey is not a donkey" [*hominis a.asinus non est asinus*] and said that the contradictory was "Of every man every donkey is c. donkey" [*Cuiuslibet hominis quilibet asinus est c.asinus*]. 'c' indicated that 'donkey' was to be treated as having merely confused supposition in relation to the first term with distributive supposition, and to be treated as having determinate supposition in relation to the second term with distributive supposition. This was a simple matter of arranging the order of descent. If a term has merely confused supposition, it is analyzed out after all the other terms, whereas terms with determinate supposition have priority. Thus, said Enzinas, from the sentence in question we first descend to "Of this man every donkey is b.donkey" [*Istius hominis quilibet asinus est b.asinus*]. Two of the rules already given are operating here, namely that terms with distributive supposition have priority over terms with merely confused supposition, and that the modifier has priority over the modifiable. In future descent, the second occurrence of 'donkey' would retain its determinate supposition. That is, it would be analyzed out before the first occurrence of 'donkey', and the sentences in which it appears would each be replaced by a disjunction of sentences which contain the singular term 'donkey_i' ($i = 1$ to n). The new overall rule was: if a term is distributed in relation to two other terms of which the first has determinate supposition and the second has merely confused supposition, and if the distributed term stands alone as subject or predicate, then in its contradictory it has the mixed supposition denoted by 'c', and the other two terms have distributive supposition. If the first of the other terms had had merely confused supposition, and the second determinate supposition, then the distributed term would be governed by 'd' in the contradictory. That is, it would be treated as determinate in relation to the first term with distributive supposition, thus having priority over it, and it would be treated as merely confused in relation to the second term with distributive supposition. Domingo de Soto added that if more than three terms were involved, then the signs 'e', 'f', and so on, could be used to indicate the order of descent. It was this proliferation of special signs that Vives, the Spanish humanist, ridiculed in his *In Pseudo Dialecticos*.[37]

Many of the same issues were raised in the discussion of another type of non-standard proposition, namely that containing a noun in nominative case, a verb, and a direct object. The favourite example was "There is a head that every man has" or, more literally, "A head has every man" [*Caput omnis homo habet*]. This form was frequently contrasted with "Every man has a head" [*Omnis homo habet caput*]; and it was pointed out that, in the present circumstances, the second is true, but the first is false, because it implies that the very same head is possessed by all men.[38] The reason for the difference was clear. 'Head' in the second sentence has merely confused supposition, but in the first sentence it has determinate supposition; and one cannot infer the first from the second without violating the basic suppositional rule mentioned earlier. So much is perfectly straightforward, but two other issues are not: that is, first, how the first sentence is to be contradicted, and second, how one is to carry out an actual analysis of the sentence, given the standard tools of supposition theory.

Domingo de Soto and Enzinas both posed the problem of contradiction in the same way.[39] In "A head every man is having", 'man' has distributive supposition in relation to 'having', which has merely confused supposition. However, it cannot have determinate supposition in the contradictory, since "Every head b.man is not having" [*Omne caput b.homo non habet*] is just as false as the original sentence, since it is only true if someone has no head at all. Again, in "A head every man is having", 'man' has distributive supposition in relation to 'head', which has determinate supposition, but it cannot have merely confused supposition in the contradictory. "A head every man is having" and "Every head some man is not having" [*Omne caput homo non habet*] would both be true if in fact all men did share a head. Domingo de Soto suggested substituting 'horse' to make this example more plausible, and he said that the proposed contradictory could be seen to be true if one descended first to "This horse a.man is not having" [*Hunc equum a.homo non habet*], then to "This horse a.man is not identical to Peter-having" [*Hunc equum a.homo non est petrus habens*] and finally to "This horse Paul is not identical to Peter-having" [*Hunc equum paulus non est petrus habens*]. The problem of contradiction was solved by using 'c' and by putting forward as the contradictory of the original sentence "Every head c.man is not having" [*Omne caput c.homo non habet*]. However, in my own view, the problem of analysis remains an intractable one. When one is descending from a genitive, one can retain the word 'of' and the relations expressed in the original sentence are not lost; but descending to such singular terms as 'Peter-having' or to 'haver$_i$' seems only to obscure the relations expressed in the original sentence. Indeed, I do not seem to be able to find any plausible analysis of the sentence in question, given only the standard suppositional rules of analysis. This problem does not seem to have struck logicians working in the medieval tradition, but it is nevertheless a real problem.

Although the logicians whose work I have examined display considerably more flexibility and subtlety than scholastic logicians have usually been credited with, their discussion reveals two important weaknesses. In the first place, they can only cope with the relations expressed in certain kinds of sentences, particularly those containing genitives; and in the second place, they do not give adequate instructions for distinguishing the case in which one is speaking of all members of a class such as donkeys from the case in which one is speaking only of the members of a subclass, such as the donkeys belonging to a particular man. On the other hand, they are clearly sensitive to the different facets of such relationships as donkey-ownership, and they are also sensitive to the kinds of inference which have to be debarred. A complete account of these strengths and weaknesses will have to await further research.

NOTES

1. I would like to thank Mrs. Rita Guerlac whose work on Vives suggested this topic to me. I would also like to thank the Canada Council for the grants which made the research for this paper possible.

2. *Cf.* P. T. Geach, *Reference and Generality*, Ithaca, New York (1962), p. 15 ff.

3. P. T. Geach, "History of a fallacy" in *Logic Matters*, Oxford (1972), pp. 1-13.

4. Domingo de Soto, *Introductiones dialectice*, Burgis (1529), xxv$^{\mathrm{vo}}$.

5. Proper descent was frequently said to involve the presence of an existential premiss. For a full discussion of this and other matters connected with descent, such as whether proper names counted as singular terms, see E. J. Ashworth, *Language and Logic in the Post-Medieval Period*, Dordrecht (1974), p. 213 ff.

6. See e.g., *Commentum in primum et quartum tractatus Petri Hyspani*, Hagennaw (1495) [no pagination], section on contradictories, condition 11.

7. Domingo de Soto, *op. cit.*, xlviii$^{\mathrm{vo}}$, wrote that 'a' and 'b' had been in use for about twenty years. The earliest reference I have discovered is in *Commentum*, where the anonymous author used 'a' as other logicians used 'b'.

8. E.g. Hieronymus of St. Mark, *Compendium preclarum*, Coloniensi (1507) [no pagination] *Questio XII*, 6th instantia; John Major, *Opera Logicalia*, Lugduni (1516), lx$^{\mathrm{vo}}$.

9. Petrus Tartaretus, *Expositio in Summulas Petri Hispani*, Parrhisiis (1520), lxxviii. The same principle was at work in the celebrated sophism "Of every man some eye is not an eye" [*Cuiuslibet hominis oculus non est oculus*] which is true because each man has two eyes. 'a' did not usually appear in the original sentence, since the first occurrence of 'man' was taken to have merely confused supposition there, but it had to appear in the descendents because without it 'eye' in "Of this man a. eye is not an eye" would have determinate supposition. See Fernando de Enzinas, *Termini*, Toletani (1533) [no pagination]; Jacobus de Naveros, *Preparatio dialectica*, Compluti (1542), xxviii$^{\mathrm{vo}}$; Domingo de Soto, *op. cit.*, xlviii.

10. Major, *loc. cit.*

11. Fernando de Enzinas, *Primus Tractatus Summularum*, Compluti (1523), xiiii$^{\mathrm{vo}}$; Hieronymus of St. Mark, *op. cit.*, *Questio I*, on descent; Naveros, *op. cit.*, xxviii$^{\mathrm{vo}}$; William Manderston, *Tripartitum in totius dialectices artis principia*, (s.l., 1530) [no pagination], on supposition; Juan Martinez Siliceo, *Primaria dyalectices elementa*, Salmantice (1517), lxiii.

12. Manderston, *loc. cit.*; Naveros, *loc. cit.*; Siliceo, *loc. cit.*; de Soto, *op. cit.*, xx.

13. de Soto, *op. cit.*, xlviii. *Cf.* Antonius Coronel, *Secunda Pars Rosarii*, Parisiis (1512), xxvi^{vo}.

14. de Soto, *op. cit.*, xlviii^{vo}, lxiv^{vo}.

15. de Soto, *op. cit.*, l^{vo} [i.e.50vo].

16. The main meaning of '*quilibet*' is 'any', but I will translate it as 'every'. No logical distinction made in the early sixteenth century will be obscured by this, and 'every' conveys the possibility of collective ownership better than does 'any'.

17. E.g. Major, *op. cit.*, cviii^{vo}; Fernando de Enzinas, *Oppositionum liber primus*, Lugduni (1528), xxxvi^{vo}; Jacques Le Fèvre d'Etaples, *Artificiales nonnulle introductiones per Judocum Clichtoveum in unum diligenter collecte*, Parisiis (1520), 25vo f.

18. Manderston, *op. cit.*, on ascent and descent; *Commentum*, on the quantity of propositions.

19. Tartaretus, *op. cit.*, lxxiv^{vo}; John Buridan, *Summula de dialectica*, Lyon (1487), [no pagination] Tract IV. This edition contains the lengthy commentary by John Dorp.

20. Buridan, *loc. cit.*

21. Albert of Saxony, *Perutilis logica*, Venetiis (1522), 29vo f.

22. It was standard practice to omit the particular sign. It should be noted that some of the words preceded by 'some' in English have merely confused rather than determinate supposition, because they are governed indirectly by a universal sign.

23. Manderston, *loc. cit.*; Coronel, *Secunda Pars*, xxvii^{vo}; de Soto, *op. cit.*, xxv^{vo} f.

24. Coronel, *loc. cit.*; Manderston, *loc. cit.* Manderston added that the rule was not necessary in the case of negative propositions.

25. Coronel, *Secunda Pars*, xxxvii^{vo}, I have supplied one clause which was omitted from I2a, namely "or of Plato Grivellus is running".

26. Buridan, *loc. cit.*

27. de Soto, *op. cit.*, l^{vo} f [i.e. 50vo f]. I find it difficult to make sense of his other claim, that "*a. hominis quilibet equus currit*" is equivalent to "*Hominis quilibet equus currit*". *Cf.* Manderston, *loc. cit.*

28. Albert of Saxony, *op. cit.*, 30; Buridan, *op. cit.*, Tract V; *Commentum*, on syllogisms, rule nine.

29. Albert of Saxony, *op. cit.*, 14vo; Antonius Coronel, *Prima Pars Rosarii*, Parisiis (1512) [no pagination], *Tercia questio de contradictoriis obliquorum*.

30. Buridan, *loc. cit.*

31. Tartaretus, *op. cit.*, lxxix^{vo} f.

32. See Coronel, *Prima Pars*, and Enzinas, *Oppositiones*. If anyone with patience and a good deal of time wishes to pursue this topic, other sources include: Robert Caubraith, *Quadrupertitum in oppositiones*, Paris (1510); Gaspar Lax, *Tractatus de oppositionibus propositionum*, Parisius (1512); and Hieronymus Pardo, *Medulla dyalectices*, Parisius (1505).

33. Coronel, *Prima Pars, loc. cit.*

34. *Cf.* Enzinas, *Oppositiones*, xxxvii f.

35. *Cf. Commentum*, on contradictories.

36. Enzinas, *Oppositiones*, xxxix$^{\mathrm{vo}}$; de Soto, *op. cit.*, xlviii.

37. J. L. Vives, *In Pseudo Dialecticos*, 44, in *Opera Omnia* III (Valencia, 1782/London, 1964) [facsimile edition].

38. *Commentum*, first question on supposition; Johannes Gebwiler, *Magistralis totium Parvuli artis logices compilatio* (Basileorum urbe, 1511) [no pagination].

39. Enzinas, *Oppositiones*, xxxix ff.; de Šoto, *loc. cit.*

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