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Anderson's Deontic Logic and Relevant Implication

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In [1], Anderson proposed that his well-known reduction schema for defining deontic operators within intensional logics should be formulated in the Anderson-Belnap logic R of relevant implication.* My purpose in this paper is to examine this proposal.

For those unfamiliar with Anderson's work in deontic logic I will provide a brief summary before turning to the main task of the paper. From 1956 onward Anderson formulated and defended the view that the logic of norms, i.e., obligations, permissions, prohibitions, and the like, should be explored by treating normative statements as certain kinds of conditionals. Thus to say of a certain act, e.g., John's closing the door, that it is *obligatory* (to say that John *ought* to close the door), is to say that if the act is not performed (John doesn't close the door), then some undesirable state-of-affairs results. Put formally, Anderson's schema captures this understanding of obligation: let O be the sentence operator "It is obligatory that," let $\Rightarrow$ be a conditional connective, and let V be a sentential constant which denotes the undesirable state-of-affairs. Then Op, when p is some sentence letter, is defined as follows:

$$Op =_{df} \sim p \Rightarrow V.$$

Many commentators on Anderson's early papers on this subject fastened upon the constant V and argued: (a) undesirable states-of-affairs do not always follow infractions, even where they are specified as in some statutes, and (b) the most general sorts of norms, in any case, cannot be understood as involving Andersonian conditionals. Too much, in my view, was made of the

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suggestion that V stood for some *bad* state-of-affairs and, as a result, these criticisms missed the essence of Anderson's view. Consider, for example, the following situation: White opens a chess game with P-KR6. Anyone who knows the rules of the game will know that such a move is forbidden, i.e., that one *ought not* to open with P-KR6. Of course this is not a rule likely to be written in any of the official rulebooks, but nevertheless it is a rule of chess and must follow from any adequate formulation of the rules of the game. But what is it to say that it is a rule of chess that one shouldn't open with P-KR6? According to Anderson, it is to say that if one opens with P-KR6, then one is not (really) playing chess. Or, expressed by the contrapositive, if one wants to (really) play chess, then one cannot open with P-KR6. Note there is no claim here that anything *bad* comes from disobeying a rule of chess; but clearly *something* comes from it, namely that one has in fact violated a rule and is not really playing the game. In the context of chess, this is all there is to V.

But what of the relationship between unfulfilled obligations and their consequences? In early formulations, Anderson believed that some necessary connection should be claimed to hold between the nonperformance of an obligation and the undesirable state-of-affairs. So he formulated the reduction schema in the modal logic S_4:

$$Op =_{df} \sim p \prec V (= \Box(\sim p \supset V)).$$

By adding this definition plus the sentential constant V to S_4 a great number of desirable deontic theorems can be proved. Along the way one must add an extra axiom to the effect that $\sim V$ is self-consistent, viz. $\sim\Box V$. I shall henceforth refer to this as *the axiom of avoidance* because the axiom states that the undesirable state-of-affairs can be avoided. If one looks at just those theorems of S_4^V (the system which results from the addition of V, the reduction schema, and the axiom of avoidance to S_4) which can be written in terms of sentence variables, truth-functional connectives, and the operator O, then this class is completely axiomatized by the following schemas, plus the rule modus ponens ([5], p. 431).

O1 *A, where A is a tautology*
O2 $OA \supset \sim O \sim A$
O3 $O(A \supset B) \supset (OA \supset OB)$
O4 $OA \supset OOA$
O5 *OA, where A is an instance of any axiom schema.*

Thus as a purely formal reduction of one logic to another, Anderson's proposal was impeccable. However, Anderson was not primarily interested in the deontic fragment of S_4^V, which could be separately developed in any case, but in S_4^V itself. And this system proved to have a number of undesirable features.

Among the theorems of S_4^V are the following:

S1 $\Box p \prec Op$
S2 $Op \prec \Box Op$
S3 $(p \prec q) \prec O(p \prec q)$.

Because of the deontic inference rule

$$\text{if } \vdash A, \text{ then } \vdash OA$$

and the modal rule which corresponds to it

$$\text{if } \vdash A, \text{ then } \vdash \Box A,$$

one is tempted to view S1 as innocuous. But these *rules* record mere formal principles that tautologies or logical truths are obligatory (necessary) which is quite different from the upshot of S1. Suppose the necessity operator $\Box$ is read as physical or causal necessity. Then S1 would make it contradictory to say that although I was compelled to do a certain act, it was wrong to do it. To see this, suppose $\Box p$ and $O \sim p$ are the case. Then by Schema O2 above, $O \sim p$ implies $\sim O \sim\sim p$, or, equivalently, $\sim Op$. By S1, $\Box p$ implies Op, and hence the contradiction. Theorem S2 makes all obligations necessary, which conflicts with the use of the term in contexts where the obligatoriness is obviously stipulated, e.g., chess. Finally, S3 gives rise to the infamous Good Samaritan paradox. Let p be "The Samaritan helps Jones who has been robbed," let q be "Jones has been robbed". Since p obviously strictly implies q, then the entire conditional is obligatory according to S3. But by O3 above, then the obligation the Samaritan has to help Jones strictly implies that there is an obligation that Jones has been robbed (i.e., from $p \prec q$ it follows by O3 and S3 that $Op \prec Oq$). Although technically interesting, S_4 plus the Anderson schema did not supply an adequate formal account of normative logic, so by 1967 Anderson had given it up.

Like the other modal systems of C. I. Lewis, S_4 was originally formulated to provide a better formal rendering of "if . . . then . . ." statements and a more perspicuous logic of implication than had been provided by material implication. Indeed, the differences between material implication and strict implication are abundantly clear when Anderson's schema is formulated by means of the two connectives. Using $\supset$ as the conditional in the definition of Op results in $\sim p \supset V$. Because of the absence of a necessity operator in truth-functional logic, the axiom of avoidance can be expressed by the formula $OA \supset \sim O \sim A$ (thus making it an axiom schema). By the definition, the equivalent formula is

$$(\sim A \supset V) \supset \sim(A \supset V).$$

But this is equivalent to $(\sim A \supset V) \supset (A \;\&\; \sim V)$. Hence, $\sim A \supset V$ implies A. However, translating back into deontic notation, this means that $Op \supset p$ is a theorem, which is clearly unacceptable. Another odd consequence of using material implication is the theoremhood of $p \supset Op$, which results from the "paradox" of material implication $p \supset (\sim p \supset V)$. Since these two indicate that $p \equiv Op$ is provable in this system, it is obvious that no deontic logic can be formulated with Anderson's schema plus material implication. But even though strict implication is an improvement over material implication, there are good reasons apart from the odd deontic logic produced by Andersonian methods to regard it as inadequate. Its fundamental shortcoming, according to Anderson [1], is that it commits *fallacies of relevance*. Such fallacies are familiar from truth-functional logic where one may infer the "conditional" $A \supset B$, from either $\sim A$ or B regardless of whether A and B have any relevance to one another. As Anderson and Belnap, [2], put it: "The archetype of fallacies of relevance is $A \Rightarrow (B \Rightarrow A)$, which would enable us to infer that Bach wrote the Coffee Cantata from the premiss that the Van Allen Belt is donut shaped—or indeed from any premiss you like." But S_4 contains theorems of the form $A \prec (B \prec A)$ (where A is of the form $C \prec D$), so it too commits such fallacies and these crop up in theorems of S_4^V. A case in point S1, $\Box p \prec (\sim p \prec V)$, is an example of a fallacy of relevance.

In the 1967 paper Anderson gave yet another reason for switching to some alternative system to S_4 as the basis of his deontic logic. He had come to believe that the relation between unfulfilled obligation and the undesirable state-of-affairs was neither logically nor causally necessary. Although there had, of course, to be *some* connection between $\sim p$ and V in the definition of Op, this connection should be such, he argued, that it could be stipulated, as in games like chess. Consequently he proposed that the system R of relevant implication which he and Belnap had formulated be used as the basis of deontic logic. The "if . . . then . . ." connective of R which I shall denote by $\rightarrow$ is meant to convey what one might call relevant material implication. In order for $A \rightarrow B$ to be true there has to be some connection between the antecedent A and the consequent B, but this connection is short of either causal or logical connection. (The system E which does contain a true logical entailment connective will be discussed below.)

In addition to the rules of adjunction and modus ponens, R has the following thirteen axiom schemas:

R1 $A \rightarrow A$
R2 $(A \rightarrow B) \rightarrow ((B \rightarrow C) \rightarrow (A \rightarrow C))$
R3 $A \rightarrow ((A \rightarrow B) \rightarrow B)$
R4 $(A \rightarrow (A \rightarrow B)) \rightarrow (A \rightarrow B)$
R5 $(A \,\&\, B) \rightarrow A$
R6 $(A \,\&\, B) \rightarrow B$
R7 $((A \rightarrow B) \,\&\, (A \rightarrow C)) \rightarrow (A \rightarrow (B \,\&\, C))$
R8 $A \rightarrow (A \vee B)$
R9 $B \rightarrow (A \vee B)$
R10 $((A \rightarrow C) \,\&\, (B \rightarrow C)) \rightarrow ((A \vee B) \rightarrow C)$
R11 $(A \,\&\, (B \vee C)) \rightarrow ((A \,\&\, B) \vee C)$
R12 $(A \rightarrow \sim B) \rightarrow (B \rightarrow \sim A)$
R13 $\sim\sim A \rightarrow A.$

I shall call the system which results from the addition of the sentential constant V, the definition

D $Op =_{df} \sim p \rightarrow V$

and the additional axiom (of avoidance) $\sim(\sim V \rightarrow V)$ to R, the *deontic logic* R^V. This formulation of the axiom of avoidance is provably equivalent to $Op \rightarrow \sim O \sim p$, so it correctly states that the sanction can be avoided, that is, that not every act leads to V.

R^V has many desirable features as a deontic logic. All of the clearly intuitive deontic schemas below are theorems of R^V:

D1	$O(A \to B) \to (OA \to OB)$
D2	$O(A \mathbin{\&} B) \to (OA \mathbin{\&} OB)$
D3	$(OA \mathbin{\&} OB) \to O(A \mathbin{\&} B)$
D4	$OA \to {\sim}O{\sim}A.$

And the deontic version of Becker's Law[1]

RO→ *If* $\vdash A \to B$, *then* $\vdash OA \to OB$

is derivable in R^V. The formula of deontic expansion $OA \to OOA$ which is a theorem of S_4^V, however, is not provable in R^V, but the perhaps more acceptable $OOA \to OA$ is.

However, R^V also has its faults. The *law of permutation*

$$(p \to (q \to r)) \to (q \to (p \to r))$$

is provable in R^V, so with obvious substitutions the following theorem results.

R^V1 $(p \to Oq) \to ({\sim}q \to O{\sim}p).$

This principle seems prima facie implausible, especially when one takes $A \to OB$ as the general form of conditional obligation in R^V. Suppose, for example, one has the obligation to lower the blinds if it isn't sunny; it would not seem to follow from this that if one doesn't lower the blinds then it is obligatory that it be sunny. Perhaps one could imagine some Olympian deity endorsing such an implication, sunny days being as much in his or her power as closing blinds, but it surely doesn't hold for mortals.

This seeming failure of R^V1 as an acceptable thesis cannot be reconciled by restricting the variables $p, q, r,$ etc., as ranging over human acts (lowering the blinds) and not over states-of-affairs in general (being sunny outside). To see this, suppose Peter is obligated to take out the garbage each evening and to put it in the can by the curb. Further suppose Peter, like everybody else in the family, is under the conditional obligation:

Anyone taking out the garbage ought to close the lid tightly on the can.

Now Peter knows (I shall assume) that he just won't remember to close the lid tightly on the can, so by means of R^V1 he concludes that he isn't even permitted to take out the garbage, inasmuch as he will fail to close the lid tightly on the can. His reasoning is as follows:

1. I ought to take out the garbage (Op)
2. But I will forget to put the lid back on (${\sim}q$)
3. If I take out the garbage, I ought to put the lid back on ($p \supset Oq$)
4. Hence, if I don't put the lid back on, I ought not to take out the garbage (${\sim}q \supset O{\sim}p$; from (3) and R^V1)
5. Hence, I ought not to take out the garbage ($O{\sim}p$; (2) + (4)).

Since all of the sentence variables in this example range over human acts, clearly R^V1 cannot be saved by the ploy of variable restriction.

Anderson explicitly discusses the effect of the permutation theorem of R on his deontic system and concludes that R^V1 is only an apparent "bad guy." In its defence he marshals the following example:

> If making a promise (p) implies that we ought to fulfill it (Oq) then if it is not to be fulfilled ($\sim q$), the promise should not have been made ($O \sim p$). And although the example is gummed up with matters having to do with the tenses of English verbs (a topic the formalism does not take into account), it still lies not too harshly on the ear. ([1], p. 358)

Although I believe Anderson is right in saying R^V1 holds for this case, the other cases I have discussed show it not to hold in general. So what accounts for the applicability of R^V1 to Anderson's example? The answer, I think, is that, in his case, there is a connection between making a promise and keeping it which is lacking in my examples. Perhaps the best way of illustrating this is by noting that the *joint prohibition*: "One shouldn't make a promise and not keep it," lies behind Anderson's example. It is characteristic of joint prohibitions that they obey the following law, where $J(p \,\&\, q)$ represents the joint prohibition of p and q:

$$J(p \,\&\, q) \Longleftrightarrow ((p \Rightarrow O \sim q) \,\&\, (q \Rightarrow O \sim p)).$$

A clear example is "Don't drink and drive!" (I take the imperative form to be a variant of the "ought" form.) From this it obviously follows–as the National Highway Safety Council would insist–both: "If you drive, don't (you ought not to) drink" and "If you drink, don't (you ought not to) drive". Conditional obligations, of course, cannot in general be converted into joint prohibitions. The principle

$$(p \Rightarrow O \sim q) \Rightarrow J(p \,\&\, q),$$

as we saw in the case of Peter and the garbage, just doesn't hold.

We seem to have found, then, two strengths of conditional obligation: a weaker variety for which R^V1 does not hold, and a stronger variety, involved in joint prohibitions, for which it does. Curiously, these two correspond to rival proposals for symbolizing conditional obligation in the early literature on deontic logic. As an appropriate notation for "Given p, it ought to be the case that q" Von Wright originally suggested $O(p \supset q)$, whereas Hintikka opted for $p \supset Oq$. Von Wright's version matches Anderson's in this sense: from "Given p, it ought to be the case that q" one can infer "Given not q, it is forbidden that p". Formally, $O(p \supset q)$ implies $O(\sim q \supset \sim p)$. Hintikka's $p \supset Oq$ does not have this feature.

From the discussion of my various examples it is evident that the stronger, Andersonian formulation of $p \rightarrow Oq$ is inadequate for general conditional obligation statements. But it does adequately represent, so far, the joint prohibition of p and not q (or, as we have seen, of q and not p). So suppose we provisionally treat Anderson's logic as holding for joint prohibitions, but not for weaker conditional obligations, in order to investigate its other features.

Other theorems of R^V, however, suggest that even in the restricted application developed in the last paragraph R^V has serious flaws. Consider the theorem

R^V2 $\quad O(p \rightarrow q) \longleftrightarrow (p \rightarrow Oq).$

This says, in effect, that there is no discernable difference in R^V between the internal conditional $O(p \rightarrow q)$ and the external $p \rightarrow Oq$. An immediate consequence of this fact is that one can detach Oq from either $p \rightarrow Oq$ together with Op, or $O(p \rightarrow q)$ together with p (the first follows from D1 plus $\mathrm{R^V}2$; the second is immediate from $\mathrm{R^V}2$). The first of these, which I list for future reference,

$\mathbf{R^V3}$ $\quad (Op \mathbin{\&} (p \rightarrow Oq)) \rightarrow Oq$

has been discussed in the literature on deontic logic because it seemed at one time a likely addition to Von Wright's early system. However, as Hintikka [3] retorted, such a principle is suited only to an ideal world in which all obligations are in fact discharged (in deontic systems with the truth-functional conditional, $Op \supset p$ is a consequence of $\mathrm{R^V}3$ (with $\supset$ for $\rightarrow$)). In terms more pertinent to this discussion $\mathrm{R^V}3$ says that if p is obligatory and p and q are jointly prohibited, then q is (categorically) prohibited. To see just how this grates on the intuitions, consider the following example:

Suppose I promise to drive Smith home from the party we are both attending, thus incurring the obligation to drive him home. Of course, drinking and driving are jointly prohibited, so if I drive Smith home, then I ought not to drink. According to $\mathrm{R^V}3$, then, from the fact that I ought to drive Smith home (inasmuch as I have so promised) together with the joint prohibition against drinking and driving, it follows that I ought not to drink. Backing up a step, from my *promising* to drive Smith home it follows that I ought not to drink (by $\mathrm{R^V}3$ and the transitivity of $\rightarrow$). But surely this is wrong, however tempting. If I do in fact drive Smith home, I ought not to drink. But from *merely promising* to drive him I seem not to incur any additional obligation. Suppose I decide to break my promise to Smith and remain at the party. If I also drink, do I violate two obligations: one by not driving and another by drinking? It seems not; the two obligations are not logically connected. I have to actually carry out my promise, to fulfill my obligation, in order also to be obligated not to drink. So $\mathrm{R^V}3$ is not an acceptable deontic theorem.

An even more troubling R^V theorem is the following:

$\mathbf{R^V4}$ $\quad OOp \rightarrow p$

(expressing OOp by means of the contraposition of Definition D gives $\sim V \rightarrow (\sim V \rightarrow p)$. This formula by permutation and contraposition yields $\sim p \rightarrow (\sim V \rightarrow V)$, which implies, because of $\vdash \sim(\sim V \rightarrow V)$, $\sim\sim p$, and, hence, p). Without pretending to have any good intuitions concerning the meaning of iterated deontic operators, I view $\mathrm{R^V}4$ as unacceptable because it permits a fact to follow from an obligation. It isn't quite as noxious as $Op \rightarrow p$ would be, but it is very close. The contraposed version of $\mathrm{R^V}4$ is $p \rightarrow PPp$ which also is unacceptable. These theorems secure what the others strongly suggested: there is no plausible interpretation of R^V which makes it an appropriate deontic logic. Theorem $\mathrm{R^V}1$ clearly indicated that the $\rightarrow$ of R^V is not adequate to satisfy the demands of the logic of conditional obligation, and when we restricted our use of R^V to joint prohibitions, we still found theorems which have no place in a satisfactory account of this fragment of normative logic. So despite its improvement over S_4^V as an Andersonian deontic logic, R^V must be rejected as well.

In order to know what conclusion to draw from all of this, it is useful to reflect on the sources of the troublesome theorems of R^V. The three theorems to which I have specifically objected, R^V1, R^V2, and R^V4, depend for their proofs on the law of permutation: $(p \rightarrow (q \rightarrow r)) \rightarrow (q \rightarrow (p \rightarrow q))$. If one thinks of R^V's difficulties as stemming from an inability of $\rightarrow$ to convey (in the presence of Anderson's schema) a contingent sense of "if . . . then . . .", it is clear that no weaker system of relevant implication is likely to repair this defect. For any such system will have the law of permutation, since truth-functional logic has it. Permutation in an Andersonian deontic logic itself will produce R^V1; with contraposition, another standard law of contingent conditionality, one gets R^V2; and by use of the avoidance axiom, R^V4 follows. These considerations coupled with the fact that R seems generally to provide a satisfactory, paradox-free sense of contingent "if . . . then . . ." suggest to me that the fault lies not with the logical base but with the Anderson schema. For in company with any conditional sentence, say,

$$p \Rightarrow q$$

the schema entails

$$Op \Rightarrow Oq$$

so long as the conditional connective is transitive (i.e., from $p \Rightarrow q$ and $q \Rightarrow r$, $p \Rightarrow r$ follows) and is subject to contraposition. From $p \Rightarrow q$, $\sim q \Rightarrow \sim p$ follows by contraposition; from $\sim q \Rightarrow \sim p$ and $\sim p \Rightarrow V(Op)$, $\sim q \Rightarrow V(Oq)$ follows by transitivity. Hence one might say that given these two fairly reasonable principles of implication, the direct upshot of Anderson's thesis concerning the foundation of obligation is the principle:

A $\quad (p \Rightarrow q) \Rightarrow (Op \Rightarrow Oq)$.

What principle A indicates is that in an Andersonian deontic logic with transitivity and contraposition there will be no way to express a contingent, conditional obligation. From "If John leaves the party now he is sure to fall down the stairs" it would follow in such a system (by means of A) that "If John ought to leave the party now, then he ought to fall down the stairs". Principle A is thus a much stronger constraint on the utility of Andersonian deontic logics than is the standard *rule* of inference RO $\rightarrow$ (if $\vdash A \rightarrow B$, then $\vdash OA \rightarrow OB$), for the rule merely records the fact that obligation carries through logical consequence, whereas A says that obligation carries through all implications. In other words, given the Andersonian thesis, all implications are treated *as if they were* logical implications.

The only logical system where A is a plausible principle is one in which the "if . . . then . . ." connective *already* stands for logical implication, i.e., systems of entailment. Perhaps the best known of these is Anderson and Belnap's system E (see [2]) which is a subsystem of R (as well as a subsystem of S_4). We can extend E to a deontic logic E^V in the now familiar manner: define Op as $\sim p \rightarrow V$ ($\rightarrow$ is henceforth the arrow of E) and add the axiom $\sim\Box V$ ($\Box A$ is definable within E as $(A \rightarrow A) \rightarrow A$). All of D1-D4 and RO $\rightarrow$ are E^V-provable, so E^V has at least the basic features of an acceptable deontic system. But E^V, unfortunately, fails to deliver the sort of theorems one might want from a logic of deontic entailment. Since entailments are matters of logical necessity (which in E is reflected by the theorem schema $(A \rightarrow B) \rightarrow \Box(A \rightarrow B))$ one would think true entailments would in some sense by obligatory. But E^V does not have

$$(p \rightarrow q) \rightarrow O(p \rightarrow q)$$

as a theorem, although S_4^V does have it for strict implication (recall S3). Furthermore, one would also expect obligatory entailments to be true, since logic sort of works that way—whatever ought to logically be, is. But again, E^V does not have among its theorems:

$$O(p \rightarrow q) \rightarrow (p \rightarrow q).$$

Strangely, however, E^V has theorems tantalizingly close to these:

E^{V}1 $\quad (p \rightarrow q) \rightarrow P(p \rightarrow q)$
E^{V}2 $\quad O \sim(p \rightarrow q) \rightarrow \sim(p \rightarrow q).$

And E^V has two other theorems familiar from S_4^V which make sense in the context of deontic entailment, *viz.*,

E^{V}3 $\quad Op \rightarrow \Box Op$
E^{V}4 $\quad Op \rightarrow \Diamond p.$

Although the Anderson schema in an entailment system looks as though it might define an operator akin to logical necessity, its role in E^V shows even this not to be the case.

The failure of S_4, R, and E to provide a satisfactory deontic logic from Anderson's schema raises the question of whether *any* logic has this capability. This general question is too large to even begin to discuss here, but I at least can provide some aids to those who want to tackle it. I have assumed throughout this paper that the basic theses of standard deontic logics, which I listed as D1-D4 and RO $\rightarrow$, are *sine qua non* of any reasonable deontic system. And, as I have shown, all of S_4^V, R^V, and E^V have them as theorems. So these seem a good place to begin if one wants to check some logic to see how it behaves as an Andersonian deontic system. The simplest way to do this checking is to use the contraposed form of the Anderson schema, which is due to Prior. Prior took $\sim V$ to mean "one escapes the sanction (undesirable state-of-affairs)" and developed escapism as the logical basis of ethics in [4]. For typographical ease he used E for $\sim V$. The Prior formulation of D1-D4 and the strong and weak deontic rules are given below.

D1 $\quad O(p \Rightarrow q) \Rightarrow (Op \Rightarrow Oq)$:
$\quad (E \Rightarrow (p \Rightarrow q)) \Rightarrow ((E \Rightarrow p) \Rightarrow (E \Rightarrow q))$
D2 $\quad O(p \,\&\, q) \Rightarrow (Op \,\&\, Oq)$:
$\quad (E \Rightarrow (p \,\&\, q)) \Rightarrow ((E \Rightarrow p) \,\&\, (E \Rightarrow q))$
D3 $\quad (Op \,\&\, Oq) \Rightarrow O(p \,\&\, q)$:
$\quad ((E \Rightarrow p) \,\&\, (E \Rightarrow q)) \Rightarrow (E \Rightarrow (p \,\&\, q))$
D4 $\quad Op \Rightarrow \sim O \sim p$:
$\quad (E \Rightarrow p) \Rightarrow \sim(E \Rightarrow \sim p).$

It should be noted that D4 in many systems requires the Axiom of Avoidance in the form $\sim(E \Rightarrow \sim E)$. Another standard law which might be added to this list records the behavior of O in company with disjunction.

D5 $(Op \vee Oq) \Rightarrow O(p \vee q)^2$:
$((E \Rightarrow p) \vee (E \Rightarrow q)) \Rightarrow (E \Rightarrow (p \vee q))$.

The two rules, RO and RO$\Rightarrow$ have the following formulations.

RO If $\vdash A$, then $\vdash OA$:
If $\vdash A$, then $\vdash E \Rightarrow A$.

RO$\Rightarrow$ If $\vdash A \Rightarrow B$, then $\vdash OA \Rightarrow OB$:
If $\vdash A \Rightarrow B$, then $\vdash (E \Rightarrow A) \Rightarrow (E \Rightarrow B)$.

NOTES

1. Named for Otto Becker who first used the modal formulation $\vdash A \supset B \Rightarrow \vdash \Box A \supset \Box B$. The stronger deontic law RO, $\vdash A \Rightarrow \vdash OA$, does not hold in R^V which is one of its strong points. So all tautologies need not count, e.g., as rules of chess.

2. The converse of D5, $O(p \vee q) \Rightarrow (Op \vee Oq)$, is generally not a desirable thesis. In standard deontic logics it would entail that both $Op \vee O\sim p$ and $(Pp \;\&\; Pq) \Rightarrow P(p \;\&\; q)$ are theorems.

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