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Research Article

Robust Simultaneous Stabilization Control Method for Two Port-Controlled Hamiltonian Systems: Controller Parameterization

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This paper investigates robust simultaneous stabilization (RSS) control method for two port-controlled Hamiltonian (PCH) systems and proposes results on the design of simultaneous stabilization controller with parameters for such systems. Firstly, two PCH systems are studied. Using the dissipative Hamiltonian structural properties, the systems are combined to generate an augmented PCH system. When there are external disturbances in the systems, a robust controller with parameters is designed for the systems. Secondly, an algorithm for solving parameters of the controller is proposed with symbolic computation. Finally, an illustrative example is presented to show that the RSS controller obtained in this paper works very well.

1. Introduction

In recent years, port-controlled Hamiltonian (PCH) systems have been extensively studied in [1–6]. Indeed, the Hamiltonian function in PCH systems is considered as the total energy (sum of potential energy and kinetic energy) in mechanical systems and is good candidate of Lyapunov functions for many physical systems. Due to this and its nice structural properties, PCH systems have drawn a good deal of attention in practical control designs. Up to now, the energy-based approach has been used in various control problems and its applications have been well investigated for a wide range of physical systems, including power systems and robotic systems. Cheng et al. [7] considered the stabilization of excitation control of power systems and proposed the model of the generalized Hamiltonian systems, which consists of externally supplied energy, dissipation, and internal energy source. Xin and Kaneda [8, 9] presented a necessary and sufficient condition for nonexistence of any singular point in the derived control law and provided a complete analysis of convergence of energy and the motion of the Acrobot.

In practical control designs, due to system's uncertainty, failure modes, or systems with various modes of operation, the simultaneous stabilization problem has often to be taken into account. The problem is concerned with designing a single controller which can stabilize all the systems simultaneously. In this way, the controller implementation cost will be greatly reduced. So far, many important results have been obtained for the simultaneous stabilization of linear systems [10–14]. In general, it is difficult to design a simultaneous stabilization controller for a class of nonlinear systems, but it is a work worth doing for many researchers [15–20]. Ho-Mock-Qai and Dayawansa [15] proposed a new method to show that, given any countable family of stabilizable nonlinear systems, there is a continuous state feedback law which simultaneously stabilizes the family. Wang et al. [16] proposed a number of results on the design of simultaneous stabilization controller for the cases of two PCH systems and more than two PCH systems. Xu et al. [17] presented sufficient conditions for simultaneous stabilization with and without H_∞ performance. Sun and Wang [18] studied simultaneous stabilization of a class of nonlinear descriptor systems via the Hamiltonian function method and proposed two new results for the simultaneous stabilization and robust simultaneous stabilization, respectively. Wei et al. [19] designed the parallel simultaneous stabilization for a set of multi-input nonlinear PCH systems with actuator saturation. Abdel-Magid et al. [20] proposed the genetic algorithms for the simultaneous stabilization of multimachine power systems over a wide range of operating conditions via single-setting power system stabilizers.

Controller parameterization is a fundamental problem in the control theory and has aroused considerable attention in recent decades. Lu et al. [21] and Isidori and Astolfi [22] proposed a family of nonlinear H_∞ controllers via output feedback. Astolfi [23] presented a family of nonlinear state-feedback controllers, in which the system state and the external disturbance are measurable. Yung et al. [24] extended the state-space formulas and presented a family of H_∞ state-feedback controllers for n-dimensional nonlinear system. Xu and Hou [25, 26] studied the generalized Hamiltonian system and proposed a family of parameterized controllers in H_∞ control and adaptive control. The controllers [21–26] are intended to solve the control problem for just one system. There are fewer works for RSS control design of two PCH systems.

Therefore, how to find a method for designing controller with parameters to solve RSS problem for two PCH systems is a challenging issue. In this paper, we investigate RSS problem for two PCH systems and present a novel, straightforward, and convenient method to design a controller with parameters to insure that two PCH systems are simultaneous stabilization. The proposed method provides support in theory for the practical application.

The remainder of this paper is organized as follows. In Section 2, the problem of RSS for PCH systems is formulated. The main contribution of this paper is then given in Section 3, in which a controller with parameters and an algorithm for solving parameters are provided, respectively. We present a numerical example for illustrating effectiveness and feasibility of controller in Section 4 and conclusions follow in Section 5.

2. Problem Formulation

Consider the following two dissipative PCH systems:

$$\sum_1 : \begin{cases} \dot{x} = [J_1(x) - R_1(x)] \dfrac{\partial H_1(x)}{\partial x} + g_1(x) u + \overline{g}_1(x)\omega, \\ y = \overline{g}_1^T(x) \dfrac{\partial H_1(x)}{\partial x}, \end{cases} \tag{1}$$

$$\sum_2 : \begin{cases} \dot{\xi} = [J_2(\xi) - R_2(\xi)] \dfrac{\partial H_2(\xi)}{\partial \xi} + g_2(\xi) u + \overline{g}_2(\xi)\omega, \\ \eta = \overline{g}_2^T(\xi) \dfrac{\partial H_2(\xi)}{\partial \xi}, \end{cases} \tag{2}$$

where $x, \xi \in \mathbb{R}^n$ and $y, \eta \in \mathbb{R}^m$ are the states vector and outputs of the two systems, respectively; $u \in \mathbb{R}^m$ is the controller with parameters; $\omega \in \mathbb{R}^s$ is the disturbance; $J_i(x) = -J_i^T(x) \in \mathbb{R}^{n\times n}$, $0 \le R_i(x) \in \mathbb{R}^{n\times n}$, $g_i(x) \in \mathbb{R}^{n\times m}$, and $\overline{g}_i(x) \in \mathbb{R}^{n\times s}$ are sufficiently smooth functions; $H_i(x)$ is the Hamiltonian function which has a local minimum at the equilibrium $x_e^{(i)}$, $i = 1, 2$, $x_e^{(1)} = x_0$, $x_e^{(2)} = \xi_0$.

Assumption 1. $H(x^{(i)}) \in C^2$ and the Hessian matrix $\mathrm{Hess}(H(x_0^{(i)})) > 0$ for systems (1) and (2).

Given a disturbance attenuation level $\gamma > 0$, choose

$$z = \Lambda \left(g_1^T(x) \frac{\partial H_1(x)}{\partial x} + g_2^T(\xi) \frac{\partial H_2(\xi)}{\partial \xi} \right) \tag{3}$$

as the penalty function, where $\Lambda \in \mathbb{R}^{s\times m}$ is a weighting matrix with full column rank and satisfies $\lambda(\Lambda^T \Lambda) \le 1$, where $\lambda(\cdot)$ denotes the eigenvalue of a matrix. Then, our objective of this section is described as follows.

RSS Control. Design an L_2 feedback controller $u = \alpha(x, \xi)$ $(\alpha(x_0, \xi_0) = 0)$, such that

(R1) the L_2 gain (from ω to z) of the closed-loop system is less than γ;

(R2) systems (1) and (2) are simultaneously asymptotically stable when ω vanishes.

hold simultaneously.

In the end, we give a definition and a lemma required in next section.

Definition 2 (see [27]). System (1) is called zero-energy-gradient (ZEG) observable with respect to y if $y(t) = 0$ and $\omega(t) = 0$, $\forall t \ge 0$, implies $\nabla H(x(t)) = 0$, $\forall t \ge 0$; system (1) is called ZEG detectable with respect to y if $y(t) = 0$ and $\omega(t) = 0$, $\forall t \ge 0$, implies $\lim_{t\to\infty} \nabla H(x(t)) = 0$; system (1) is called generalized ZEG observable (detectable) if $y(t) = 0$, $z(t) = 0$, and $\omega(t) = 0$, $\forall t \ge 0$, implies $\nabla H(x(t)) = 0$, $\forall t \ge 0$ $(\lim_{t\to\infty} \nabla H(x(t)) = 0)$.

Lemma 3 (see [27]). *Consider a nonlinear system*

$$\begin{aligned} \dot{x} &= f(x) + g(x)\omega, \qquad f(x_0) = 0 \\ z &= h(x), \end{aligned} \tag{4}$$

where $x \in \mathbb{R}^n$ is the state vector, $\omega \in \mathbb{R}^s$ is the disturbances, $z \in \mathbb{R}^q$ is the penalty. If there exists the function $V(x) \ge 0$ $(V(x_0) = 0)$, such that HJI inequality

$$\begin{aligned} &\left(\frac{\partial V}{\partial x}\right)^T f(x) + \frac{1}{2\gamma^2} \left(\frac{\partial V}{\partial x}\right)^T g(x) g(x)^T \left(\frac{\partial V}{\partial x}\right) \\ &\quad + \frac{1}{2} h(x)^T h(x) \le 0 \end{aligned} \tag{5}$$

holds, it is implied that the L_2 gain of the closed-loop system (2) *(from ω to z) is bounded by γ ($\gamma > 0$); that is,*

$$\int_0^T \|z\|^2 dt \le \int_0^T \gamma^2 \|\omega\|^2 dt. \tag{6}$$

3. Main Results

In this section, we propose an H_∞ controller with parameters for systems (1) and (2) and an algorithm for solving parameters. The parameterization methods suggest a framework to solve the RSS control problem of two PCH systems.

3.1. RSS of Two PCH Systems

Theorem 4. *Considering systems* (1) *and* (2), *with the penalty function* (3) *and the given level* $\gamma > 0$, *assume that systems* (1) *and* (2) *are generalized ZEG detectable (when* $\omega = 0$*). If*

(i) *there exists a symmetric matrix* $K \in \mathbb{R}^{m\times m}$, *satisfying* $K(\Lambda^T\Lambda + (1/\gamma^2)I_m) = (\Lambda^T\Lambda + (1/\gamma^2)I_m)K$, *such that*

$$\begin{aligned}\widetilde{R}_1(x) &= R_1(x) + K_{11}(x,x) - \frac{1}{2\gamma^2}\overline{g}_1(x)\overline{g}_1^T(x) \\ &\quad - \frac{1}{2}g_1(x)\Lambda^T\Lambda g_1^T(x) \geq 0, \\ \widetilde{R}_2(\xi) &= R_2(\xi) - K_{22}(\xi,\xi) - \frac{1}{2\gamma^2}\overline{g}_2(\xi)\overline{g}_2^T(\xi) \\ &\quad - \frac{1}{2}g_2(\xi)\Lambda^T\Lambda g_2^T(\xi) \geq 0,\end{aligned} \tag{7}$$

where $K_{ij}(x,\xi) = (1/2)g_i(x)K(\Lambda^T\Lambda + (1/\gamma^2))g_j^T(\xi)$, $i, j = 1, 2$;

(ii)

$$g_1 g_2^T = 0, \qquad \overline{g}_1\overline{g}_2^T = 0; \tag{8}$$

(iii)

$$\left[\frac{\partial H_1^T(x)}{\partial x}g_1(x) + \frac{\partial H_2^T(\xi)}{\partial \xi}g_2(\xi)\right]\Phi(x,\xi) \leq 0 \tag{9}$$

hold simultaneously, then

$$\begin{aligned}u &= \frac{1}{2}K\left(\Lambda^T\Lambda + \frac{1}{\gamma^2}I_m\right) \\ &\quad \times \left(-g_1^T(x)\frac{\partial H_1(x)}{\partial x} + g_2^T(\xi)\frac{\partial H_2(\xi)}{\partial \xi}\right) + \Phi(x,\xi)\end{aligned} \tag{10}$$

is an L_2 *disturbance attenuation controller, such that both R1 and R2 hold simultaneously for systems* (1) *and* (2), *where* $\Phi(x,\xi) \in \mathbb{R}^{m\times 1}$ *and* I_m *is an* $m \times m$ *unit matrix.*

Proof. Substituting controller (10) into systems (1) and (2), we obtain the following closed-loop systems:

$$\begin{aligned}\dot{x} &= [J_1(x) - R_1(x)]\frac{\partial H_1(x)}{\partial x} \\ &\quad + g_1(x)\left[\frac{1}{2}K\left(\Lambda^T\Lambda + \frac{1}{\gamma^2}I_m\right)\right. \\ &\qquad \times \left(-g_1^T(x)\frac{\partial H_1(x)}{\partial x} + g_2^T(\xi)\frac{\partial H_2(\xi)}{\partial \xi}\right) \\ &\qquad \left. + \Phi(x,\xi)\right] + \overline{g}_1(x)\omega, \\ \dot{\xi} &= [J_2(\xi) - R_2(\xi)]\frac{\partial H_2(\xi)}{\partial \xi} \\ &\quad + g_2(\xi)\left[\frac{1}{2}K\left(\Lambda^T\Lambda + \frac{1}{\gamma^2}I_m\right)\right. \\ &\qquad \times \left(-g_1^T(x)\frac{\partial H_1(x)}{\partial x} + g_2^T(\xi)\frac{\partial H_2(\xi)}{\partial \xi}\right) \\ &\qquad \left. + \Phi(x,\xi)\right] + \overline{g}_2(\xi)\omega.\end{aligned} \tag{11}$$

From systems (1) and (2), system (11) and the penalty function (3) can be rewritten as an augmented PCH system (12):

$$\begin{aligned}\dot{X} &= \left[\overline{J}(X) - \overline{R}(X)\right]\frac{\partial \overline{H}(X)}{\partial X} + G(X)v + \overline{G}(X)\omega \\ &= f(X) + \overline{G}(X)\omega, \\ Y &= M(X)\frac{\partial \overline{H}(X)}{\partial X}, \\ z &= \Lambda G^T(X)\frac{\partial \overline{H}(X)}{\partial X} := h(X),\end{aligned} \tag{12}$$

where $X = [x^T, \xi^T]^T$, $Y = [y^T, \eta^T]^T$, $\overline{H}(X) = H_1(x) + H_2(\xi)$, $v = \Phi(x,\xi)$

$$\begin{aligned}\frac{\partial \overline{H}(X)}{\partial X} &= \begin{bmatrix}\dfrac{\partial H_1(x)}{\partial x} \\ \dfrac{\partial H_2(\xi)}{\partial \xi}\end{bmatrix}, \\ \overline{J}(X) &= \begin{bmatrix} J_1(x) & K_{12}(x,\xi) \\ -K_{21}(\xi,x) & J_2(\xi)\end{bmatrix}, \\ \overline{R}(X) &= \begin{bmatrix} R_1(x) + K_{11}(x,x) & 0 \\ 0 & R_2(\xi) - K_{22}(\xi,\xi)\end{bmatrix}, \\ G(X) &= \begin{bmatrix} g_1(x) \\ g_2(\xi)\end{bmatrix}, \qquad \overline{G}(X) = \begin{bmatrix}\overline{g}_1(x) \\ \overline{g}_2(\xi)\end{bmatrix}, \\ M(X) &= \begin{bmatrix}\overline{g}_1^T(x) & 0 \\ 0 & \overline{g}_2^T(\xi)\end{bmatrix}.\end{aligned} \tag{13}$$

Consider the candidate Lyapunov function $V(X) = \overline{H}(X) - c \geq 0$, where $c = \overline{H}(X_0)$. With Lemma 3 and the conditions of the theorem, we have

$$\begin{aligned}
&\left(\frac{\partial V}{\partial X}\right)^T f(X) + \frac{1}{2\gamma^2}\left(\frac{\partial V}{\partial X}\right)^T \overline{G}(X)\overline{G}^T(X)\left(\frac{\partial V}{\partial X}\right) \\
&\quad + \frac{1}{2}h^T(X)h(X) \\
&= -\left(\frac{\partial \overline{H}}{\partial X}\right)^T \overline{R}(X)\frac{\partial \overline{H}}{\partial X} \\
&\quad + \left(\frac{\partial \overline{H}}{\partial X}\right)^T G(X)\nu + \frac{1}{2\gamma^2}\left(\frac{\partial \overline{H}}{\partial X}\right)^T \overline{G}(X)\overline{G}^T(X)\left(\frac{\partial \overline{H}}{\partial X}\right) \\
&\quad + \frac{1}{2}h^T(X)h(X) \\
&= -\left[\nabla H_1^T \ \ \nabla H_2^T\right] \\
&\quad \times \begin{bmatrix} R_1 + K_{11} & 0 \\ 0 & R_2 - K_{22} \end{bmatrix}\begin{bmatrix} \nabla H_1 \\ \nabla H_2 \end{bmatrix} \\
&\quad + \left[\nabla H_1^T \ \ \nabla H_2^T\right]\begin{bmatrix} g_1 \\ g_2 \end{bmatrix}\Phi(x,\xi) \\
&\quad + \frac{1}{2\gamma^2}\left[\nabla H_1^T \ \ \nabla H_2^T\right]\begin{bmatrix} \overline{g}_1 \\ \overline{g}_2 \end{bmatrix}\left[\overline{g}_1^T \ \ \overline{g}_2^T\right]\begin{bmatrix} \nabla H_1 \\ \nabla H_2 \end{bmatrix} \\
&\quad + \frac{1}{2}\left[\nabla H_1^T \ \ \nabla H_2^T\right]\begin{bmatrix} g_1 \\ g_2 \end{bmatrix}\Lambda^T\Lambda\left[g_1^T \ \ g_2^T\right]\begin{bmatrix} \nabla H_1 \\ \nabla H_2 \end{bmatrix} \\
&= -\left(\nabla H_1^T(R_1 + K_{11})\nabla H_1 + \nabla H_2^T(R_2 - K_{22})\nabla H_2\right) \\
&\quad + \left(\nabla H_1^T g_1 + \nabla H_2^T g_2\right)\Phi(x,\xi) \\
&\quad + \frac{1}{2\gamma^2}\left(\nabla H_1^T\overline{g}_1\overline{g}_1^T\nabla H_1 + \nabla H_2^T\overline{g}_2\overline{g}_1^T\nabla H_1\right. \\
&\qquad \left. + \nabla H_1^T\overline{g}_1\overline{g}_2^T\nabla H_2 + \nabla H_2^T\overline{g}_2\overline{g}_2^T\nabla H_2\right) \\
&\quad + \frac{1}{2}\left(\nabla H_1^T g_1\Lambda^T\Lambda g_1^T\nabla H_1 + \nabla H_2^T g_2\Lambda^T\Lambda g_1^T\nabla H_1\right. \\
&\qquad \left. + \nabla H_1^T g_1\Lambda^T\Lambda g_2^T\nabla H_2 + \nabla H_2^T g_2\Lambda^T\Lambda g_2^T\nabla H_2\right) \\
&= -\nabla H_1^T\left(R_1 + K_{11} - \frac{1}{2\gamma^2}\overline{g}_1\overline{g}_1^T - \frac{1}{2}g_1\Lambda^T\Lambda g_1^T\right)\nabla H_1 \\
&\quad - \nabla H_2^T\left(R_2 - K_{22} - \frac{1}{2\gamma^2}\overline{g}_2\overline{g}_2^T - \frac{1}{2}g_2\Lambda^T\Lambda g_2^T\right)\nabla H_2 \\
&\quad + \left(\nabla H_1^T g_1 + \nabla H_2^T g_2\right)\Phi(x,\xi) \leq 0.
\end{aligned} \tag{14}$$

According to the lemma, the L_2 gain of system (12) (from ω to z) is no more than γ and R1 holds.

Next, we prove that system (12) is asymptotically stable when $\omega = 0$. When $\omega = 0$, it is easy to know from (12) that

$$\begin{aligned}
\dot{V}(X) &= \left(\frac{\partial V}{\partial X}\right)^T\left[\overline{J}(X) - \overline{R}(X)\right]\left(\frac{\partial V}{\partial X}\right) + \left(\frac{\partial V}{\partial X}\right)^T G(X)\nu \\
&= -\left(\frac{\partial \overline{H}(X)}{\partial X}\right)^T \overline{R}(X)\left(\frac{\partial \overline{H}(X)}{\partial X}\right) \\
&\quad + \left(\frac{\partial \overline{H}(X)}{\partial X}\right)^T G(X)\nu \\
&= -\left[\nabla H_1^T \ \ \nabla H_2^T\right]\begin{bmatrix} R_1 + K_{11} & 0 \\ 0 & R_2 - K_{22} \end{bmatrix} \\
&\quad \times \begin{bmatrix} \nabla H_1 \\ \nabla H_2 \end{bmatrix} + \left[\nabla H_1^T \ \ \nabla H_2^T\right]\begin{bmatrix} g_1 \\ g_2 \end{bmatrix}\Phi(x,\xi) \\
&= -\nabla H_1^T\left(R_1 + K_{11} - \frac{1}{2\gamma^2}\overline{g}_1\overline{g}_1^T - \frac{1}{2}g_1\Lambda^T\Lambda g_1^T\right)\nabla H_1 \\
&\quad - \frac{1}{2\gamma^2}\nabla H_1^T\overline{g}_1\overline{g}_1^T\nabla H - \frac{1}{2}\nabla H_1^T g_1\Lambda^T\Lambda g_1^T\nabla H \\
&\quad - \nabla H_2^T\left(R_2 - K_{22} - \frac{1}{2\gamma^2}\overline{g}_2\overline{g}_2^T - \frac{1}{2}g_2\Lambda^T\Lambda g_2^T\right)\nabla H_2 \\
&\quad - \frac{1}{2\gamma^2}\nabla H_2^T\overline{g}_2\overline{g}_2^T\nabla H_2 - \frac{1}{2}\nabla H_2^T g_2\Lambda^T\Lambda g_2^T\nabla H_2 \\
&\quad + \left(\nabla H_1^T g_1 + \nabla H_2^T g_2\right)\Phi(x,\xi) \leq 0.
\end{aligned} \tag{15}$$

Thus, the solution of the closed-loop system converges to the largest invariant set contained in

$$\begin{aligned}
S &:= \left\{X : \dot{V}(X) = 0\right\} \\
&\subset \left\{X : y = \overline{g}_1^T\nabla H_1 \equiv 0, \eta = \overline{g}_2^T\nabla H_2 \equiv 0,\right. \\
&\qquad g_1^T\nabla H_1 \equiv 0, g_2^T\nabla H_2 \equiv 0, \\
&\qquad \left. \forall t \geq 0\right\}.
\end{aligned} \tag{16}$$

From the fact that system (12) is generalized ZEG detectable, we know that $\overline{g}_1^T\nabla H_1 \equiv 0$, $g_1^T\nabla H_1 \equiv 0 \Rightarrow x \to x_0$ $(t \to \infty)$, and $\overline{g}_2^T\nabla H_2 \equiv 0$, $g_2^T\nabla H_2 \equiv 0 \Rightarrow \xi \to \xi_0$ $(t \to \infty)$. Hence, the largest invariant set contains only one point; that is, $X_0 = [x_0^T, \xi_0^T]^T$ which is the equilibrium point. From LaSalle's invariance principle, the closed-loop system (12) is asymptotically stable at its equilibrium and R2 holds. This completes the proof. □

Remark 5. (1) Conditions (7) and (8) in Theorem 4 are not restrictive and can be easily satisfied in many systems.

(2) $\Phi(x,\xi)$ is a polynomial vector with parameters. We can obtain the parameters of $\Phi(x,\xi)$ via solving condition (9).

(3) The proposed parameterization method can be used for a nonlinear control system, and of course the first step

in applying the method is to express the nonlinear system as a dissipative Hamiltonian system based on dissipative Hamiltonian realization methods [28, 29].

(4) The current studies of the proposed parameterization method merely remain in theory, but it will be used for practical applications such as multimachine power systems [20], because of the broad applicability of the method for nonlinear control system.

3.2. Solving Parameters Algorithm (SP). From condition (7), we can obtain the γ^*. Let $\gamma \geq \gamma^*$ such that condition (7) holds. Then we propose an algorithm to find parameters ranges of controller (10) via solving the parameters of $\Phi(x,\xi)$ in condition (9). The SP algorithm now proceeds as follows.

(S1) Set $\Phi(x,\xi) = [\Phi_1(x), \Phi_2(\xi)]^T$ and suppose a positive integer r, which is the degree of polynomial vector $\Phi(x,\xi)$. Write $\Phi_i(x_i) = \sum_{j=1,r}^{j=l} a_{ij} p_r(x_i)$, where $l = \sum_r c(n+r-1,r)$, $p_r(x) = \prod_{i=1}^{n} x_i^{r_i}$, and n is the number of state variable.

(S2) Let $S = -[(\partial H_1^T(x)/\partial x) g_1(x) + (\partial H_2^T(\xi)/\partial \xi) g_2(\xi)]\Phi(x,\xi)$.

(S3) The influence of high order items can be ignored because this paper considers locally asymptotically stable for system. Choose all terms of $\deg(S) \geq 3$ and $\deg(S) = 1$ from S and let the coefficients of these terms be zero. So obtain a set of equations A.

(S3.1) Observe equations A. Let some parameters be zero and substitute them into A. Then obtain the simplified equations A'.

(S3.2) Obtain a set of parameters solution U_1 via solving A' by using cylindrical algebraic decomposition (CAD) algorithm [30].

(S3.3) Substitute U_1 into S and obtain a new polynomial S, which is a quadratic form.

(S4) Rewrite S as coefficient matrix M, and all principal minors of M must be positive semidefinite [31]. Choose all principal minors of M and obtain inequalities B.

(S4.1) Observe inequalities B. Let some parameters be zero and substitute them into B. Then obtain the simplified inequalities B'.

(S4.2) Obtain a set of parameters solution U_2 via solving B' by using CAD algorithm.

(S5) Let $U = U_1 \cup U_2$ and substitute U into controller (7) and thus obtain the polynomial controller with parameters. This completes the algorithm.

Remark 6. (1) The SP algorithm starts from $r = 1$ normally.

(2) The CAD algorithm is given by semialgebraic-set-tools of regular-chains in Maple 16.

(3) It is merely to simplify computation that we let some parameters be zero before using CAD algorithm. However, these parameters are not necessarily zero. So the set of parameters solution obtained by SP algorithm is a subset of solutions.

4. Numerical Experiments

Consider the following PCH systems with external disturbances described as

$$J_1(x) = \begin{bmatrix} 0 & x_1 & 0 \\ -x_1 & 0 & -x_2 \\ 0 & x_2 & 0 \end{bmatrix}, \qquad g_1 = \begin{bmatrix} 1 & -1 \\ 0 & 0 \\ -1 & 1 \end{bmatrix},$$

$$\overline{g}_1 = \begin{bmatrix} 1 & 1 \\ 0 & 0 \\ 1 & 1 \end{bmatrix},$$

$$R_1(x) = \operatorname{Diag}\{3,1,3\}, \qquad H_1(x) = \frac{1}{2}\left(x_1^2 + 2x_2^2 + x_3^2\right),$$

$$J_2(\xi) = \begin{bmatrix} 0 & -\xi_2 & 2\xi_3 \\ \xi_2 & 0 & 0 \\ -2\xi_3 & 0 & 0 \end{bmatrix}, \qquad g_2 = \begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 0 & 0 \end{bmatrix},$$

$$\overline{g}_2 = \begin{bmatrix} 1 & -1 \\ -1 & 1 \\ 0 & 0 \end{bmatrix},$$

$$R_2(\xi) = \operatorname{Diag}\{5,5,5\}, \qquad H_2(\xi) = \frac{1}{2}\left(\xi_1^2 + \xi_2^2 + \xi_3^2\right). \tag{17}$$

4.1. Controller Design and Solving Parameters. From system (17), it is easy to get

$$\operatorname{Hess}(H(x_0)) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} > 0, \tag{18}$$

$$\operatorname{Hess}(H(\xi_0)) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} > 0. \tag{19}$$

So the assumption holds.

Given a disturbance attenuation level $\gamma > 0$, choose

$$z = \Lambda\left(g_1^T \nabla H_1 + g_2^T \nabla H_2\right). \tag{20}$$

Let $K = \left[\begin{smallmatrix} 1 & 0 \\ 0 & -1 \end{smallmatrix}\right]$, $\Lambda = \left[\begin{smallmatrix} \sqrt{3}/2 & 0 \\ 0 & \sqrt{3}/2 \end{smallmatrix}\right]$.
Then, $K^T K = \left[\begin{smallmatrix} 1 & 0 \\ 0 & 1 \end{smallmatrix}\right]$, $\Lambda^T \Lambda = \left[\begin{smallmatrix} 3/4 & 0 \\ 0 & 3/4 \end{smallmatrix}\right]$,

$$K\left(\Lambda^T \Lambda + \frac{1}{\gamma^2} I_m\right) = \left(\Lambda^T \Lambda + \frac{1}{\gamma^2} I_m\right) K = \begin{bmatrix} \frac{3}{4} + \frac{1}{\gamma^2} & 0 \\ 0 & -\frac{3}{4} - \frac{1}{\gamma^2} \end{bmatrix}. \tag{21}$$

A straightforward computation shows that when $\gamma \geq 2$

$$\widetilde{R}_1(x) = \begin{bmatrix} \frac{9}{4} - \frac{1}{\gamma^2} & 0 & \frac{3}{4} - \frac{1}{\gamma^2} \\ 0 & 1 & 0 \\ \frac{3}{4} - \frac{1}{\gamma^2} & 0 & \frac{9}{4} - \frac{1}{\gamma^2} \end{bmatrix} \geq 0,$$

$$\widetilde{R}_2(\xi) = \begin{bmatrix} \frac{17}{4} - \frac{1}{\gamma^2} & -\frac{3}{4} + \frac{1}{\gamma^2} & 0 \\ -\frac{3}{4} + \frac{1}{\gamma^2} & \frac{17}{4} - \frac{1}{\gamma^2} & 0 \\ 0 & 0 & 5 \end{bmatrix} \geq 0, \tag{22}$$

$$g_1 g_2^T = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \qquad \overline{g}_1 \overline{g}_2^T = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

From system (17), we have $[-g_1^T \nabla H_1 + g_2^T \nabla H_2] = [-x_1 + x_3 + \xi_1 + \xi_2 \quad x_1 - x_3 + \xi_1 + \xi_2]^T$ and obtain that

$$u = \begin{bmatrix} \left(\frac{3}{8} + \frac{1}{2\gamma^2}\right)(-x_1 + x_3 + \xi_1 + \xi_2) \\ \left(-\frac{3}{8} - \frac{1}{2\gamma^2}\right)(x_1 - x_3 + \xi_1 + \xi_2) \end{bmatrix} + \Phi(x, \xi). \tag{23}$$

Let $\Phi(x, \xi) = \left[\begin{smallmatrix} \Phi_1(x) \\ \Phi_2(\xi) \end{smallmatrix}\right]$. We know $n = 6$ in system (12) and let $r = 1$. We have $\Phi_1(x) = a_1 x_1 + a_2 x_2 + a_3 x_3$, $\Phi_2(\xi) = b_1 \xi_1 + b_2 \xi_2 + b_3 \xi_3$, where a_i, b_i, $i = 1, 2, 3$, are the parameters.

From system (17), we obtain that $[\nabla H_1^T g_1 + \nabla H_2^T g_2] = [x_1 - x_3 + \xi_1 + \xi_2 \quad -x_1 + x_3 + \xi_1 + \xi_2]$.

Let $S = -[\nabla H_1^T g_1 + \nabla H_2^T g_2] \Phi(x, \xi)$, and we have

$$\begin{aligned} S = {} & -a_1 x_1^2 - a_2 x_1 x_2 - (a_3 - a_1) x_1 x_3 - (a_1 - b_1) x_1 \xi_1 \\ & - (a_1 - b_2) x_1 \xi_2 + b_3 x_1 \xi_3 + a_2 x_2 x_3 \\ & - a_2 x_2 \xi_1 - a_2 x_2 \xi_2 + a_3 x_3^2 - (a_3 + b_1) x_3 \xi_1 \\ & - (a_3 + b_2) x_3 \xi_2 - b_3 x_3 \xi_3 - b_1 \xi_1^2 \\ & - (b_1 + b_2) \xi_1 \xi_2 - b_3 \xi_1 \xi_3 - b_2 \xi_2^2 - b_3 \xi_2 \xi_3. \end{aligned} \tag{24}$$

S is a quadratic form and can be rewritten as a coefficient matrix (multiply constant 2 for simplifying computation):

$$M = \begin{bmatrix} -2a_1 & -a_2 & -a_3 + a_1 & -a_1 + b_1 & -a_1 + b_2 & b_3 \\ -a_2 & 0 & a_2 & -a_2 & -a_2 & 0 \\ -a_3 + a_1 & a_2 & 2a_3 & -a_3 - b_1 & -a_3 - b_2 & -b_3 \\ -a_1 + b_1 & -a_2 & -a_3 - b_1 & -2b_1 & -b_1 - b_2 & -b_3 \\ -a_1 + b_2 & -a_2 & -a_3 - b_2 & -b_1 - b_2 & -2b_2 & -b_3 \\ b_3 & 0 & -b_3 & -b_3 & -b_3 & 0 \end{bmatrix}. \tag{25}$$

All principal minors of M must be positive semidefinite. We have inequalities B from M. From B, we can easily obtain that $a_1 \leq 0$, $a_3 \geq 0$. Substitute $U_1 = \{a_2 = 0, b_3 = 0\}$ into inequalities B to simplify computation; we obtain simplified inequalities B'. Solving inequalities B' by using CAD algorithm, we obtain a series of sets. Choose some sets, which satisfy inequalities B', and organize them. We have

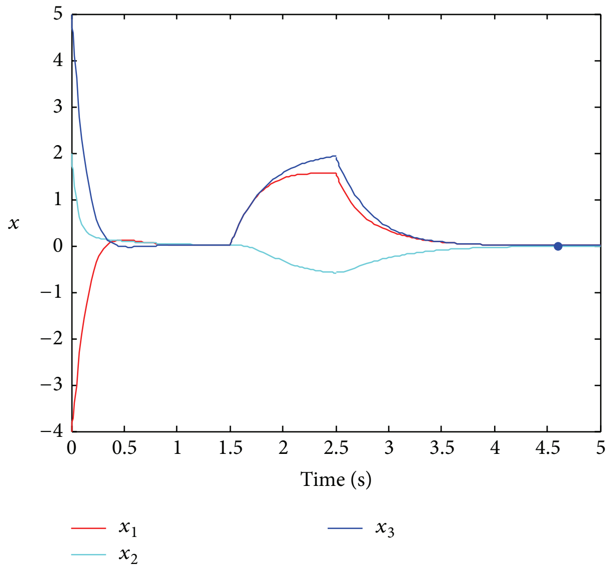

FIGURE 1: Swing curves of x.

$$U = \{a_3 = -a_1, b_1 = a_1, b_2 = a_1\} \cup U_1. \tag{26}$$

Substitute U into controller (23):

$$u = \begin{bmatrix} \left(\frac{3}{8} + \frac{1}{2\gamma^2}\right)(-x_1 + x_3 + \xi_1 + \xi_2) \\ \left(-\frac{3}{8} - \frac{1}{2\gamma^2}\right)(x_1 - x_3 + \xi_1 + \xi_2) \end{bmatrix} + \begin{bmatrix} a_1 x_1 - a_1 x_3 \\ a_1 \xi_1 + a_1 \xi_2 \end{bmatrix}, \tag{27}$$

where $a_1 \leq 0$.

So we have the controller with parameters for system (17). The controller (27) has a rather simple form.

4.2. Simulations and Results. In order to evaluate the robustness of controller (27), we set the parameters of system (17) as $\gamma = \sqrt{5}$ and the parameters of controller as $a_1 = -1$. We obtain the controller as follows:

$$u = \begin{bmatrix} \frac{59}{40}(-x_1 + x_3) + \frac{19}{40}(\xi_1 + \xi_2) \\ \frac{19}{40}(-x_1 + x_3) - \frac{59}{40}(\xi_1 + \xi_2) \end{bmatrix}. \tag{28}$$

To illustrate the effectiveness of controller (28), we carry out some numerical simulations with the following choices: $x(0) = [-4, 2, 5]^T$, $\xi(0) = [3, 4, -2]^T$. To test the robustness of the controller with respect to external disturbances, a square disturbance $\omega = [2, 4]^T$ is added to systems in the time duration $[1.5 \sim 2.5\,s]$. The simulation results are shown in Figures 1, 2, and 3, which are the responses of the state and control signal, respectively.

From Figures 1, 2, and 3, we know that controller (28) is very effective in simultaneously stabilization systems (1) and (2).

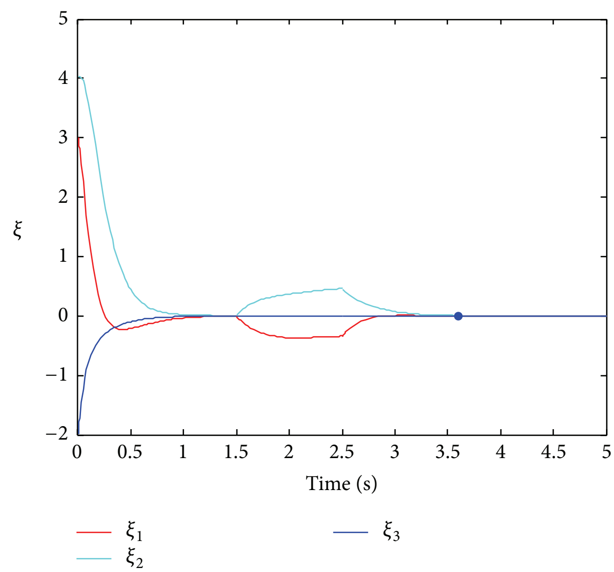

FIGURE 2: Swing curves of ξ.

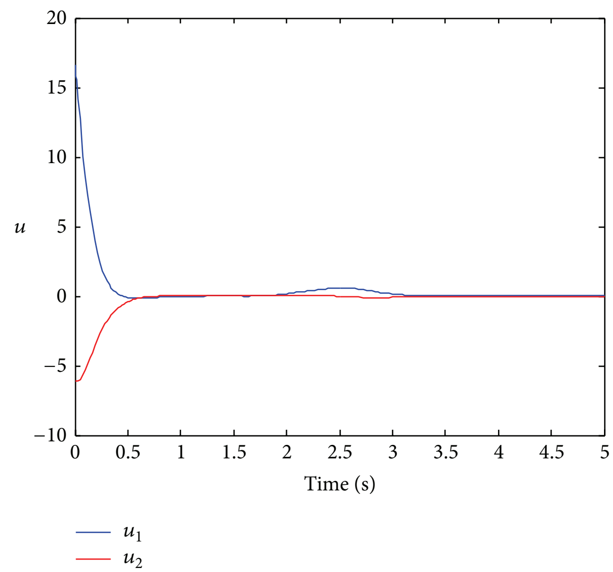

FIGURE 3: Swing curves of u.

5. Conclusions

In this paper, we have investigated the RSS problem for two PCH systems and proposed a RSS controller with parameters design method. A controller with parameters has been obtained using Hamiltonian function method and an algorithm for solving parameters of the controller has been proposed with symbolic computation. The study of illustrative example with simulations has shown that the RSS controller obtained in this paper has been efficient in H_∞ control for two PCH systems.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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