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Research Article

Complete Controllability of Impulsive Stochastic Integrodifferential Systems in Hilbert Space

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This paper concerns the complete controllability of the impulsive stochastic integrodifferential systems in Hilbert space. Based on the semigroup theory and Burkholder-Davis-Gundy's inequality, sufficient conditions of the complete controllability for impulsive stochastic integro-differential systems are established by using the Banach fixed point theorem. An example for the stochastic wave equation with impulsive effects is presented to illustrate the utility of the proposed result.

1. Introduction

It is well known that controllability is one of the fundamental concepts and plays an important role in control theory and engineering. The problem which is about controllability of linear and nonlinear stochastic systems represented by SODE (stochastic ordinary differential equation) in finite dimensional space has been extensively studied (e.g., [1–4] and references therein). The controllability for infinite dimensional stochastic systems represented by SPDE (stochastic partial differential equation) is natural generalization of stochastic systems in finite dimensional space [5]. According to the literature, at least three types of infinite dimensional stochastic systems have been studied, that is, approximate, complete, and S-controllability [6], so the controllability research of the infinite dimensional stochastic systems is usually more complicated than that of the finite dimensional. For linear stochastic system, the controllability problem has been studied by some authors [6, 7], which is shown as the following SPDE:

$$dx(t) = [Ax(t) + Bu(t)]\,dt + \Sigma(t)\,dW(t), \quad t \in [0,T],$$
$$x(0) = x_0 \in \mathscr{L}_2(\Omega, H), \tag{1}$$

where x_0 is $\mathscr{F}_0$-measurable, H is separable Hilbert space, A is the infinitesimal generator of a strongly continuous semigroup $S(t)$ on H, $B \in \mathscr{L}(U,H)$, $u(t)$ is feedback control, $W(t)$ is Q-Wiener process, and $\Sigma \in \mathscr{L}_2(Q^{1/2}E, H)$. For nonlinear stochastic systems in infinite dimensional space, there are also many results on the controllability theory (see [8–13]).

On the other hand, the impulsive effects exist widely in many evolution processes in which the states are changed abruptly at certain moments of time, involving fields such as finance, economics, mechanics, electronics, and telecommunications (see [14] and references of therein). Impulsive differential systems have emerged as an important area investigation in applied sciences, and many papers have been published about the controllability of impulsive differential systems both in finite and infinite dimensional space. Sakthivel et al. [15] established the sufficient conditions for approximate controllability of nonlinear impulsive differential systems by Schauder's fixed point theorem; Li et al. [16] investigated the complete controllability of the first-order impulsive functional differential systems in Banach space using Schaefer's fixed point theorem; Chang [17] studied the complete controllability of impulsive functional differential systems with infinite delay; Sakthivel et al. [18] discussed complete controllability of second-order nonlinear impulsive differential systems. However, the complete controllability problem of impulsive stochastic integro-differential systems has not been investigated in infinite dimensional space yet, to the best of our knowledge, although [19–22], respectively, investigated the controllability of impulsive stochastic control systems in finite dimensional space by using contraction mapping principle; and Subalakshmi and Balachandran [23] studied the approximate controllability of nonlinear stochastic impulsive systems in Hilbert spaces by using Nussbaum's fixed point theorem. Based on Banach fixed point theorem, the proposed work in this paper on the complete controllability of the integro-differential stochastic systems with impulsive effects in Hilbert spaces is new in the literature.

In this paper, our main purpose is to show the complete controllability of following impulsive stochastic integro-differential systems in Hilbert space,

$$\begin{aligned} dx(t) = &\left[Ax(t) + Bu(t)\right.\\ &\left.+ F\left(t, x(t), \int_0^t f(t,s,x(s))\,ds\right)\right] dt\\ &+ G\left(t, x(t), \int_0^t g(t,s,x(s))\,ds\right) dw(t), \quad (2)\\ &\qquad t \neq t_k,\ t \geqslant 0,\\ \Delta x(t_k) = &\ I_k\left(x_{t_k^-}\right), \quad t = t_k,\ k = 1,2,\ldots,m,\\ x(0) = &\ x_0 \in H, \end{aligned}$$

where $F : [0,T] \times H \times H \to H$, $G : [0,T] \times H \times H \to \mathscr{L}_2(Q^{1/2}E, H)$, $f, g : [0, T \times [0,T] \times H \to H$ are measurable mappings. $I_k(x_{t_k^-}) = x(t_k^+) - x(t_k^-)$, $t = t_k$, $k = 1,2,\ldots,\rho$, where $x(t_k^+)$ and $x(t_k^-)$ denote the right and left limits of $x(t)$ at $t = t_k$, respectively. Also $\Delta x(t_k) = x(t_k^+) - x(t_k^-)$ represents the jump in the state x at time t_k with I_k determining the size of the jump. For systems (2), if $I_k = 0$, the controllability problem was studied by Subalakshmi et al. [11]. If $I_k \neq 0$ and $G = 0$, $f = 0$, [15] discussed the approximate controllability problem. When A, B are matrices with appropriate dimensions, F, G are vectors (in fact, matrix is aspecial form of operator), and $f = g = 0$, Karthikeyan et al. [19] obtained the controllability results, so system (2) is of the more general form and has great diversity.

The outline of this paper is as follows: Section 2 contains basic notations, lemmas, and preliminary facts. The controllability results are given in Section 3 by fixed point methods. In Section 4, we provide an example to demonstrate the effectiveness of our method. Finally, conclusions are given in Section 5.

2. Preliminaries

Let $(\Omega, \mathscr{F}, \mathbb{P})$ be a complete probability space with a filtration $\{\mathscr{F}_t\}_{t\geqslant 0}$ satisfying the usual conditions (i.e., it is right continuous and $\mathscr{F}_0$ contains all $\mathbb{P}$-null sets). We consider three Hilbert spaces E, H, and U, and a Q-Wiener process on $(\Omega, \mathscr{F}, \mathbb{P})$ with the covariance operator $Q \in \mathscr{L}(E)$ such that $\operatorname{tr} Q < \infty$. Let $\langle\cdot\rangle$ and $\|\cdot\|$ denote inner product and norm of H, respectively. $\mathscr{L}(X,Y)$ is the space of all linear bounded operator from a Hilbert space X to a Hilbert space Y. We also employ the same notation $\|\cdot\|$ for the norm of $\mathscr{L}(X,Y)$. We assume that there exists a complete orthonormal $\{e_k\}$ in E, a bounded sequence of nonnegative real numbers λ_k such that $Qe_k = \lambda_k e_k$, $k = 1,2,\ldots$, and a sequence $\{\beta_k\}$ of independent Brownian motions such that

$$\langle w(t), e\rangle = \sum_{k=1}^{\infty} \sqrt{\lambda_k}\,\langle e_k, e\rangle\,\beta_k(t), \quad e \in E,\ t \in [0,T], \quad (3)$$

and $\mathscr{F}_t = \mathscr{F}_t^{\beta}$, where $\mathscr{F}_t^{\beta}$ is the σ-algebra generated by $\{\beta(s) : 0 \leqslant s \leqslant t\}$. Let $\mathscr{L}_2^0 = \mathscr{L}_2(Q^{1/2}E, H)$ be the space of all Hilbert-Schmidt operator from $Q^{1/2}E$ to H with the inner product $\langle \Psi, \Phi\rangle_{\mathscr{L}_2^0} = \operatorname{tr}[\Psi Q \Phi^*]$ and the norm $\|\cdot\|_{\mathscr{L}_2^0}$. $L_2(\mathscr{F}_T, H)$ is the Hilbert space of all $\mathscr{F}_T$-measurable square integrable random variables with values in Hilbert space H. $L_2^{\mathscr{F}_T}([0,T], H)$ is the Hilbert space of square integrable and $\mathscr{F}_T$-adapted processes with values in H.

Let $\mathrm{PC}([0,T], L_2(\Omega, \mathscr{F}, \mathbb{P}; H)) = \{\phi : \phi$ is a function from $[0,T]$ into $L_2(\Omega, \mathscr{F}, \mathbb{P}; H)$ such that $\phi(t)$ is continuous at $t \neq t_k$, left continuous at $t = t_k$, and the right limit $\phi(t_k^+)$ exists for $k = 1,2,\ldots,\rho\}$. $\mathscr{H}_2(U_2)$ is the closed subspace of $\mathrm{PC}([0,T], L_2(\Omega, \mathscr{F}, \mathbb{P}; H))$ consisting of measurable and $\mathscr{F}_t$-adapted H-valued(U-valued) process $\phi(\cdot) \in \mathrm{PC}([0,T], L_2(\Omega, \mathscr{F}, \mathbb{P}; H))(\phi(\cdot) \in \mathrm{PC}([0,T], L_2(\Omega, \mathscr{F}, \mathbb{P}; U)))$ endowed with the norm $\|\phi\|_{\mathscr{H}_2}^2 = \sup_{0\leqslant t\leqslant T} \mathbb{E}\|\phi(t)\|^2$.

By a solution of system (2), we mean a mild solution of the following nonlinear integral equation:

$$\begin{aligned} x(t) = &\ S(t)x_0 + \int_0^t S(t-s)Bu(s)\,ds\\ &+ \int_0^t S(t-s)F\left(s, x(s), \int_0^s f(s,\tau,x(\tau))\,d\tau\right) ds\\ &+ \int_0^t S(t-s)G\left(s, x(s), \int_0^s g(s,\tau,x(\tau))\,d\tau\right) dw(s)\\ &+ \sum_{k=1}^{\rho} S(t-t_k)I_k(x(t_k^-)), \end{aligned} \quad (4)$$

where $u \in U_{ad} := U_2$, $S(t)_{t\geqslant 0}$ denotes the strongly continuous semigroup generated by the operator A.

Now let us introduce the controllability operator Π_s^T associated with (4) (see[8]),

$$\Pi_s^T\{\cdot\} = \int_s^T S(T-t)BB^*S^*(T-t)\,dt, \quad (5)$$

which belongs to $\mathscr{L}(H,H)$; B^* is the adjoint operator of B.

Definition 1. System (2) is completely controllable on $[0,T]$ if

$$\mathscr{R}_T(x_0) = L_2^{\mathscr{F}_T}([0,T], H). \quad (6)$$

That is, all the points in $L_2^{\mathscr{F}_T}([0,T],H)$ can be reached from the point x_0 at time T, where $\mathscr{R}_t(x_0) = \{x(t;x_0,u) : u \in L_2^{\mathscr{F}}([0,T],H)\}$.

Lemma 2 (Burkholder-Davis-Gundy's inequality [23]). *For any $r \geqslant 1$ and for arbitrary $\mathscr{L}_2^0$-valued predictable process $\Psi(t)$, $t \in [0,T]$, one has*

$$\mathbb{E}\left(\sup_{0\leqslant t\leqslant T}\left|\int_0^t \Psi(s)\,dw(s)\right|^{2r}\right) \leqslant C_r \mathbb{E}\left(\int_0^t \|\Psi(s)\|^2_{\mathscr{L}_2^0} ds\right)^r, \quad (7)$$

where

$$C_r = (r(2r-1))^r \left(\frac{2r}{2r-1}\right)^{2r^2}. \quad (8)$$

Lemma 3 (Mahmudov [6]). *The following linear system*

$$dx(t) = [Ax(t) + Bu(t)]\,dt + D(t)\,dw(t), \quad x(0) = x_0 \quad (9)$$

is completely controllable if and only if $\Pi_0^T \geqslant \gamma I$, where γ is constant and I is unit operator.

Lemma 4. *Assume that the operator Π_s^T is invertible. Then for arbitrary target $x_T \in L_2(\mathscr{F}_T, H)$, the control*

$$\begin{aligned} u(t) = {} & B^* S^*(T-t)\,\mathbb{E} \\ & \times \left\{ \left(\Pi_0^T\right)^{-1} \left[x_T - S(T)x_0 - \int_0^T S(T-s)\overline{F}(s)\,ds \right.\right. \\ & \qquad - \int_0^T S(T-s)\overline{G}(s)\,dw(s) \\ & \qquad \left.\left. - \sum_{k=1}^{\rho} S(T-t_k) I_k(x(t_k^-)) \right] \mid \mathscr{F}_t \right\} \end{aligned} \quad (10)$$

transfers the systems (4) from x_0 to x_T at time T, where $\overline{F}(s) = F(s,x(s),\int_0^s f(s,\tau,x(\tau))d\tau)$, $\overline{G}(s) = G(s,x(s),\int_0^s g(s,\tau,x(\tau))d\tau)$).

Proof. Substituting (10) into (4), we can obtain that

$$\begin{aligned} x(t) = {} & S(t)x_0 + \int_0^t S(t-s) \\ & \times BB^* S^*(T-s)\,\mathbb{E} \\ & \times \left\{ \left(\Pi_0^T\right)^{-1} \left[x_T - S(T)x_0 - \int_0^T S(T-s)\overline{F}(s)\,ds \right.\right. \\ & \qquad - \int_0^T S(T-s)\overline{G}(s)\,dw(s) \\ & \qquad \left.\left. - \sum_{k=1}^{\rho} S(T-t_k) I_k(x(t_k^-)) \right] \mid \mathscr{F}_t \right\} \\ & + \int_0^t S(T-s)\overline{F}(s)\,ds \\ & + \int_0^t S(T-s)\overline{G}(s)\,dw(s) + \sum_{k=1}^{\rho} S(T-t_k) I_k(x(t_k^-)) \\ = {} & S(t)x_0 + \Pi_0^t \left\{ S^*(T-t)\left(\Pi_0^T\right)^{-1} \right. \\ & \times \left[x_T - S(T)x_0 - \int_0^T S(T-s)\overline{F}(s)ds \right. \\ & \qquad - \int_0^t S(t-s)\overline{G}(s)\,dw(s) \\ & \qquad \left.\left. - \sum_{k=1}^{\rho} S(T-t_k) I_k(x(t_k^-)) \right] \right\} \\ & + \int_0^t S(T-s)\overline{F}(s)\,ds + \int_0^t S(T-s)\overline{G}(s)\,dw(s) \\ & + \sum_{k=1}^{\rho} S(T-t_k) I_k(x(t_k^-)). \end{aligned} \quad (11)$$

The proof is completed by letting $t = T$ in (11). □

3. Main Results

In this section, by using contraction mapping principle in Banach space we discuss the complete controllability criteria of semilinear impulsive stochastic systems (2). For the proof of the main result we impose the following assumptions on data of the problem.

Assumption A. The functions F, G, and I are continuous and satisfy the usual linear growth condition; that is, there exist positive real constants L_1, α_k for arbitrary $x \in H$, and $0 \leqslant t \leqslant T$ such that

$$\begin{gathered} \|F(t,x,y)\|^2 + \|G(t,x,y)\|^2_{\mathscr{L}_2^0} \leqslant L_1\left(1 + \|x\|^2 + \|y\|^2\right), \\ \|I_k(x)\|^2 \leqslant \alpha_k\left(1 + \|x\|^2\right), \quad k = 1,2,\ldots,\rho, \\ \left\|\int_0^t f(t,s,x(s))ds\right\|^2 + \left\|\int_0^t g(t,s,x(s))ds\right\|^2 \leqslant k_1\|x\|^2. \end{gathered} \quad (12)$$

Assumption B. The functions $F, G,$ and I satisfy the following Lipschitz condition and for every $t \geqslant 0$ and $x, y \in H$ there exist positive real constants L_2, β_k, k_2 such that

$$\begin{aligned} & \|F(t,x_1,y_1) - F(t,x_2,y_2)\|^2 \\ & \quad + \|G(t,x_1,y_1) - G(t,x_2,y_2)\|^2_{\mathscr{L}_2^0} \\ & \leqslant L_2\left(\|x_1 - x_2\|^2 + \|y_1 - y_2\|^2\right), \\ & \|I_k(x) - I_k(y)\| \leqslant \beta_k\|x - y\|, \quad k = 1,2,\ldots,\rho, \end{aligned}$$

$$\begin{aligned}\int_0^t \Big\{ & \|f(t,s,x(s)) - f(t,s,y(s))\|^2 \\ & + \int_0^t \|g(t,s,x(s)) - g(t,s,y(s))\|^2 \Big\} ds \\ & \leqslant k_2 \|x-y\|^2.\end{aligned} \tag{13}$$

Assumption C. The linear system (9) is completely controllable. By Lemma 3, for some $\gamma > 0$, $\mathbb{E}\langle \Pi_0^T z, z\rangle \geqslant \gamma \mathbb{E}\|z\|^2$, for all $z \in L_2(\mathcal{F}_T, H)$. Consequently, $\|(\Pi_0^T)^{-1}\| \leqslant 1/\gamma = l_2$.

Assumption D. Let $p = [6Tl_1L_2(Ml_1l_2+1)(T+4)(1+k_2T) + 6l_1\rho(Ml_2+1)\sum_{k=1}^{\rho}\beta_k]$ be such that $0 \leqslant p < 1$.

Now for convenience, let us introduce the following notations:

$$l_1 = \max_{0\leqslant t\leqslant T}\|S(t)\|^2, \qquad M = \max_{s\in[0,T]}\left\|\Pi_s^T\right\|^2. \tag{14}$$

Theorem 5. *Suppose that assumptions A, B, C, and D are satisfied. Then system* (4) *is completely controllable on* $[0,T]$.

Proof. For arbitrary initial data $x_0 \in \mathcal{H}_2$, we can define a nonlinear operator Φ from H_2 to H_2 as the following:

$$\begin{aligned}(\Phi x)(t) = S(t)x_0 & + \int_0^t S(t-s)Bu(s)\,ds \\ & + \int_0^t S(t-s)\overline{F}(s,x)\,ds \\ & + \int_0^t S(t-s)\overline{G}(s,x)\,dw(s) \\ & + \sum_{k=1}^{\rho} S(t-t_k)I_k(x(t_k^-)),\end{aligned} \tag{15}$$

where $u(t)$ is defined by (10).

By Lemma 4, the control (10) transfers system (4) from the initial state x_0 to the final state x_T provided that the operators Φ has a fixed point in $\mathcal{H}_2$. So, if the operator Φ has a fixed point then system (2) is completely controllable. As mentioned before, to prove the complete controllability of the system (2), it is enough to show that Φ has a fixed point in $\mathcal{H}_2$. To do this, we can employ the contraction mapping principle. In the following, we will divide the proof into two steps.

Firstly, we show that $\mathcal{H}_2$ maps $\mathcal{H}_2$ into itself. From (15) we have

$$\begin{aligned}\sup_{0\leqslant t\leqslant T}\mathbb{E}\|(\Phi x)(t)\|^2 = \sup_{0\leqslant t\leqslant T}\mathbb{E}\Big\| & S(t)x_0 + \int_0^t S(t-s)Bu(s)\,ds \\ & + \int_0^t S(t-s)\overline{F}(s,x)\,ds \\ & + \int_0^t S(t-s)\overline{G}(s,x)\,dw(s) \\ & + \sum_{k=1}^{\rho} S(t-t_k)I_k(x(t_k^-))\Big\|^2 \\ \leqslant\ & 5\sup_{0\leqslant t\leqslant T}\mathbb{E}\|S(t)x_0\|^2 \\ & + 5\sup_{0\leqslant t\leqslant T}\mathbb{E}\left\|\int_0^t S(t-s)Bu(s)ds\right\|^2 \\ & + 5\sup_{0\leqslant t\leqslant T}\mathbb{E}\left\|\int_0^t S(t-s)\overline{F}(s,x)ds\right\|^2 \\ & + 5\sup_{0\leqslant t\leqslant T}\mathbb{E}\left\|\int_0^t S(t-s)\overline{G}(s,x)dw(s)\right\|^2 \\ & + 5\sup_{0\leqslant t\leqslant T}\mathbb{E}\left\|\sum_{k=1}^{\rho} S(t-t_k)I_k(x(t_k^-))\right\|^2 \\ \triangleq\ & \sum_{i=1}^{5} A_i.\end{aligned} \tag{16}$$

Using Holder inequality, B-D-G inequality (here $C_1 = 4$), and Assumption C, we have the following estimates:

$$\begin{aligned} A_1 &\leqslant 5l_1\mathbb{E}\|x_0\|^2, \\ A_3 &\leqslant 5T\sup_{0\leqslant t\leqslant T}\mathbb{E}\int_0^t \left\|S(t-s)\overline{F}(s,x)\right\|^2 ds \\ &\leqslant 5Tl_1L_1\int_0^T\left(1 + (1+k_1)\sup_{0\leqslant s\leqslant T}\mathbb{E}\|x(s)\|^2\right)ds, \\ A_4 &\leqslant 20l_1L_1\int_0^T\left(1 + (1+k_1)\sup_{0\leqslant s\leqslant T}\mathbb{E}\|x(s)\|^2\right)ds, \\ A_5 &\leqslant 5\rho\sup_{0\leqslant t\leqslant T}\mathbb{E}\sum_{k=1}^{\rho}\|S(t-t_k)I_k(x(t_k^-))\|^2 \\ &\leqslant 5\rho l_1\sum_{i-1}^{\rho}\alpha_k\left(1 + \sup_{0\leqslant t\leqslant T}\mathbb{E}\|x(s)\|^2\right).\end{aligned} \tag{17}$$

Meanwhile by control function (15), we have

$$\begin{aligned} A_2 = 5\sup_{0\leqslant t\leqslant T}\mathbb{E} & \\ \times \Big\| \int_0^t & S(t-r)BB^*S^*(T-r)\mathbb{E} \\ & \times\Big\{\left(\Pi_0^T\right)^{-1}\times\Big[x(T) - S(T)x_0 \\ & \qquad - \int_0^T S(T-s)\overline{F}(s,x)\,ds\end{aligned}$$

$$
\begin{aligned}
&\qquad - \int_0^T S(T-s)\overline{G}(s,x)\,dw(s) \\
&\qquad - \sum_{k=1}^{\rho} S(T-t_k) I_k(x(t_k^-))\Big]\Big\}\,dr\Bigg\|^2 \\
&= 5\sup_{0\leqslant t\leqslant T}\mathbb{E}\Bigg\|\int_0^t S(t-r)BB^*S^*(t-r)S^*(T-t)\left(\Pi_0^T\right)^{-1} \\
&\quad\times\Big[x(T) - S(T)x_0 - \int_0^T S(T-s)\overline{F}(s,x)\,ds \\
&\quad - \int_0^T S(T-s)\overline{G}(s,x)\,dw(s) \\
&\quad - \sum_{k=1}^{\rho} S(T-t_k) I_k(x(t_k^-))\Big]\,dr\Bigg\|^2 \\
&= 5\sup_{0\leqslant t\leqslant T}\mathbb{E}\Bigg\|\Pi_0^t S^*(T-t)\left(\Pi_0^T\right)^{-1} \\
&\quad\times\Big[x_T - S(T)x_0 - \int_0^T S(T-s)\overline{F}(s)\,ds \\
&\quad - \int_0^t S(t-s)\overline{G}(s,x)\,dw(s) \\
&\quad - \sum_{k=1}^{\rho} S(T-t_k) I_k(x(t_k^-))\Big]\Bigg\|^2.
\end{aligned}
\tag{18}
$$

So similar as in (17), we get

$$
\begin{aligned}
A_2 \leqslant 25Ml_1l_2\Bigg(&\mathbb{E}\|x_T\|^2 + l_1\|x_0\|^2 \\
&+ Tl_1L_1\int_0^T\left(1 + (1+k_1)\sup_{0\leqslant s\leqslant T}\|x(s)\|^2\right)ds \\
&+ 4l_1L_1\int_0^T\left(1 + (1+k_1)\sup_{0\leqslant s\leqslant T}\|x(s)\|^2\right)ds \\
&+ \rho\sum_{i=1}^{\rho}\alpha_k\left(1 + \sup_{0\leqslant t\leqslant T}\|x(t)\|^2\right)\Bigg).
\end{aligned}
\tag{19}
$$

From (17)–(19), we have

$$
\begin{aligned}
&\sup_{0\leqslant t\leqslant T}\mathbb{E}\|\Phi x(t)\|^2 \\
&\quad\leqslant C\left[1 + \int_0^T\left(1 + (1+k_1)\sup_{0\leqslant s\leqslant T}\mathbb{E}\|x(s)\|^2\right)ds\right] \\
&\quad\leqslant C\left[1 + \left(1 + (1+k_1)T\sup_{0\leqslant s\leqslant T}\mathbb{E}\|x(s)\|^2\right)\right]
\end{aligned}
\tag{20}
$$

for all $t \in [0,T]$, where C is constant. This implies that Φ maps $\mathscr{H}_2$ into itself.

Secondly, we prove that Φ is a contraction mapping on $\mathscr{H}_2$, for any $x, y \in \mathscr{H}_2$,

$$
\begin{aligned}
&\sup_{0\leqslant t\leqslant T}\mathbb{E}\|(\Phi x)(t) - (\Phi y)(t)\|^2 \\
&\quad= \sup_{0\leqslant t\leqslant T}\mathbb{E}\Bigg\|\Pi_0^t S^*(T-t)\left(\Pi_0^T\right)^{-1} \\
&\qquad\times\Bigg[\int_0^T S(T-s)\left(\overline{F}(s,y) - \overline{F}(s,x)\right)ds \\
&\qquad + \int_0^T S(T-s)\left(\overline{G}(s,y) - \overline{G}(s,x)\right)dw(s) \\
&\qquad + \sum_{k=1}^{\rho} S(T-t_k)\left(I_k(y(t_k^-)) - I_k(x(t_k^-))\right)\Bigg] \\
&\qquad + \int_0^t S(t-s)\left(\overline{F}(s,x) - \overline{F}(s,y)\right)ds \\
&\qquad + \int_0^t S(t-s)\left(\overline{G}(s,x) - \overline{G}(s,y)\right)dw(s) \\
&\qquad + \sum_{k=1}^{\rho} S(t-t_k)\left(I_k(x(t_k^-)) - I_k(y(t_k^-))\right)\Bigg\|^2 \\
&\quad\leqslant 6Ml_1l_2\Bigg\{\left\|\int_0^T S(T-s)\left(\overline{F}(s,x) - \overline{F}(s,y)\right)ds\right\|^2 \\
&\qquad + \left\|\int_0^T S(T-s)\left(\overline{G}(s,x) - \overline{G}(s,y)\right)ds\right\|^2 \\
&\qquad + \left\|\sum_{k=1}^{\rho} S(T-t_k)\left(I_k(x(t_k^-)) - I_k(y(t_k^-))\right)\right\|^2\Bigg\} \\
&\qquad + 6\sup_{0\leqslant t\leqslant T}\mathbb{E}\left\|\int_0^t S(t-s)\left(\overline{F}(s,x) - \overline{F}(s,y)\right)ds\right\|^2 \\
&\qquad + 6\sup_{0\leqslant t\leqslant T}\mathbb{E}\left\|\int_0^t S(t-s)\left(\overline{G}(s,x) - \overline{G}(s,y)\right)dw(s)\right\|^2 \\
&\qquad + 6\sup_{0\leqslant t\leqslant T}\mathbb{E}\left\|\sum_{k=1}^{\rho} S(t-t_k)\left(I_k(x(t_k^-)) - I_k(y(t_k^-))\right)\right\|^2 \\
&\quad\triangleq 6Ml_1l_2\sum_{i=1}^{3}B_i + \sum_{i=4}^{6}B_i.
\end{aligned}
\tag{21}
$$

Using Lipschitz condition, similiar to A_1–A_5, we have the following estimates:

$$
B_1 \leqslant Tl_1L_2(1+k_2T)\int_0^T\sup_{0\leqslant s\leqslant T}\mathbb{E}\|x(s) - y(s)\|^2 ds, \tag{22}
$$

$$B_2 \leqslant 4l_1L_2\left(1+k_2T\right)\int_0^T \sup_{0\leqslant s\leqslant T}\mathbb{E}\|x(s)-y(s)\|^2 ds, \quad (23)$$

$$B_3 \leqslant l_1\rho\sum_{k=1}^{\rho}\beta_k \sup_{0\leqslant s\leqslant T}\mathbb{E}\|x(s)-y(s)\|^2, \quad (24)$$

$$B_4 \leqslant 6Tl_1L_2\left(1+k_2T\right)\int_0^T \sup_{0\leqslant s\leqslant T}\mathbb{E}\|x(s)-y(s)\|^2 ds, \quad (25)$$

$$B_5 \leqslant 24l_1L_2\left(1+k_2T\right)\int_0^T \sup_{0\leqslant s\leqslant T}\mathbb{E}\|x(s)-y(s)\|^2 ds, \quad (26)$$

$$B_6 \leqslant 6l_1\rho\sum_{k=1}^{\rho}\beta_k \sup_{0\leqslant s\leqslant T}\mathbb{E}\|x(s)-y(s)\|^2 \quad (27)$$

together with inequalities (22)–(27):

$$\begin{aligned}
&\sup_{0\leqslant t\leqslant T}\mathbb{E}\|(\Phi x)(t)-(\Phi y)(t)\|^2 \\
&\quad\leqslant 6TMl_1^2l_2L_2\left(1+k_2T\right)\int_0^T \sup_{0\leqslant s\leqslant T}\mathbb{E}\|x(s)-y(s)\|^2 ds \\
&\qquad + 24Ml_1^2l_2L_2\left(1+k_2T\right)\int_0^T \sup_{0\leqslant s\leqslant T}\mathbb{E}\|x(s)-y(s)\|^2 ds \\
&\qquad + \left(6Ml_1l_2\rho\sum_{k=1}^{\rho}\beta_k + 6l_1\rho\sum_{k=1}^{\rho}\beta_k\right)\sup_{0\leqslant t\leqslant T}\mathbb{E}\|x(t)-y(t)\|^2 \\
&\quad\leqslant \Bigg[6Tl_1L_2\left(Ml_1l_2+1\right)(T+4)\left(1+k_2T\right) \\
&\qquad + 6l_1\rho\left(Ml_2+1\right)\sum_{k=1}^{\rho}\beta_k\Bigg]\sup_{0\leqslant t\leqslant T}\mathbb{E}\|x(t)-y(t)\|^2.
\end{aligned} \quad (28)$$

Theorefore, Φ is a contraction mapping from $\mathscr{H}_2$ to $\mathscr{H}_2$, and hence Φ has a unique fixed point. Thus the system (4) is completely controllability on $[0,T]$. □

4. Example

Consider the impulsive stochastic integrao-differential wave equation with control $u(t,z)\in L^2[0,1]$,

$$\begin{aligned}
\frac{\partial^2 z}{\partial t^2} &= a\frac{\partial^2 z}{\partial \xi^2} + bu(t,z) + F\left(t,z(t),\int_0^t f(t,s,z(s))\,ds\right) \\
&\quad + G\left(t,z(t),\int_0^t g(t,s,z(s))\,ds\right)\frac{\partial w}{\partial t}, \\
&\qquad t\neq t_k,\ 0\leqslant \xi\leqslant 1,
\end{aligned} \quad (29)$$

where $\partial w/\partial t$ is white noise and initial and boundary conditions are

$$\begin{gathered}
z(t,0)=z(t,1)=0, \quad t\neq t_k, \\
\Delta z(t_k)(\xi) = I_k^1\left(z_{t_k}\right), \qquad \Delta z'(t_k)(\xi) = I_k^2\left(z_{t_k}\right), \\
z(0,\xi)=z_0(\xi), \qquad \frac{\partial z(0,\xi)}{\partial t} = z_1(\xi).
\end{gathered} \quad (30)$$

Let $H=L^2[0,1]$; then $A: H\to H$, $Az=z''$. Domain of operator A is

$$\begin{aligned}
D(A) = \{&z\in H \mid z,\ z' \text{ is continuous}, z''\in H, \\
&z(0)=z(1)=0\}.
\end{aligned} \quad (31)$$

Let

$$Z = \begin{bmatrix} z \\ \dfrac{\partial z}{\partial t}\end{bmatrix}, \qquad Z(0) = \begin{bmatrix} z_0 \\ z_1\end{bmatrix}, \qquad B = \begin{bmatrix} 0 \\ b\end{bmatrix}, \quad (32)$$

$$\begin{aligned}
\overline{F} &= \begin{bmatrix} 0 \\ F\left(t,z(t),\int_0^t f(t,s,z(s))\,ds\right)\end{bmatrix}, \\
\overline{G} &= \begin{bmatrix} 0 \\ G\left(t,z(t),\int_0^t g(t,s,z(s))\,ds\right)\end{bmatrix}.
\end{aligned} \quad (33)$$

Then the system (29) is

$$\begin{gathered}
dZ = \left[\mathscr{A}Z + Bu + \overline{F}(t,Z)\right]dt + \overline{G}(t,Z)\,dW, \quad t\neq t_k, \\
\Delta Z_k = \begin{bmatrix} I_k^1 \\ I_k^2\end{bmatrix}, \quad t=t_k,\ k=1,2,\ldots,m,\ Z(0) = \begin{bmatrix} z_0 \\ z_1\end{bmatrix},
\end{gathered} \quad (34)$$

where

$$\mathscr{A} = \begin{bmatrix} 0 & I \\ -A & 0\end{bmatrix}, \quad (35)$$

and $\mathscr{A}$ is the infinitesimal generator of a strongly continuous semigroup $h(t)$, $t\geqslant 0$, on $X=D(A^{1/2})\oplus H$, for $x\times y\in X$:

$$S(t)\begin{bmatrix} x \\ y\end{bmatrix} = \sum_{n=1}^{\infty}\begin{bmatrix} \cos(n\pi t) & (n\pi)^{-1}\sin(n\pi t) \\ -(n\pi)\sin(n\pi t) & \cos(n\pi t)\end{bmatrix}\begin{bmatrix} x_n \\ y_n\end{bmatrix}e_n, \quad (36)$$

$e_n(\theta)=\sqrt{2}\sin(n\pi\theta)$, $\theta\in[0,1]$. Write $h(t)$ is

$$h(t) = \begin{bmatrix} C(t) & S(t) \\ AS(t) & C(t)\end{bmatrix}, \quad (37)$$

where

$$S(t)x := \int_0^t C(s)x\,ds, \quad x\in H. \quad (38)$$

The mild solution of system (29) is

$$z(t) = C(t) z_0 + S(t) z_1 + \int_0^t S(t-s)\left[Bu(s) + f(s,z)\right] ds + \int_0^t S(t-s) g(s,z)\, dw_s + \sum_{0\leqslant t_k \leqslant t} C(t-t_k) I_k^1\left(z_{t_k}\right) + \sum_{0\leqslant t_k\leqslant t} S(t-t_k) I_k^2\left(z_{t_k}\right) \quad (39)$$

by [7]; the stochastic linear system of (29) is complete controllable. Then from Theorem 5 one can easily prove system (29) is completely controllable, if the functions F, G, f, g, I_k^1, I_k^2 satisfy Lipschitz condition and linear growth condition.

5. Conclusions

The complete controllability of impulsive stochastic integro-differential systems in Hilbert space has been investigated in this paper. Sufficient conditions of complete controllability for impulsive stochastic integro-differential systems are established by using the Banach fixed point theorem. An example illustrates the efficiency of proposed results.

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