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INVERSE LIMITS OF INDECOMPOSABLE CONTINUA

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Let $\{X_\lambda, f_{\lambda\mu}, \Lambda\}$ denote an inverse limit system of continua, with inverse limit space X_∞. Capel has shown that if each X_λ is an arc (simple closed curve), then X_∞ is an arc (simple closed curve) provided that Λ is countable and the bonding maps are monotone and onto. It is shown in this paper that a similar result holds when each X_λ is a pseudo-arc. In fact, the restrictions that the bonding maps be monotone and onto may be deleted.

Two theorems are proved which lead to this result. First, it is shown that if the maps of an inverse system of indecomposable continua are onto, then the limit space is an indecomposable continuum. Next, it is shown that with no restrictions on the bonding maps, a similar statement is true for hereditarily indecomposable continua.

1. Definitions and notation. All spaces are assumed to be Hausdorff. The notation $\{X_\lambda, f_{\lambda\mu}, \Lambda\}$ represents an inverse limit system with factor spaces X_λ, bonding maps $f_{\lambda\mu}$ and directed set Λ. The inverse limit space of the system $\{X_\lambda, f_{\lambda\mu}, \Lambda\}$ is denoted by X_∞. Definitions of these terms may be found in [2]. For each $\lambda \in \Lambda$, Π_λ denotes the projection function of $P_{\lambda\in\Lambda} X_\lambda$ onto X_λ, restricted to X_∞.

A *continuum* is a compact connected Hausdorff space. A continuum is *indecomposable* if it cannot be expressed as the union of two proper subcontinua. It is *hereditarily indecomposable* if each of its subcontinua is indecomposable.

A *chain* is a finite collection of open sets $U_1, \cdots, U_n$ such that $U_i \cap U_j \neq \emptyset$ if and only if $|i - j| \leqq 1$. A space X is said to be *chainable* if each open covering of X has a chain refinement. Hence a chainable space is a continuum.

If X is a metric space and $U_1, \cdots, U_n$ is a chain covering of X such that for some $\varepsilon > 0$, diameter $U_i < \varepsilon$ for $i = 1, \cdots, n$, then the chain $U_1, \cdots, U_n$ is said to be an *ε-chain covering* of X. A metric space X is *snakelike* if for each $\varepsilon > 0$, there exists an ε-chain covering of X.

2. Preliminary results. The following basic results will be needed. When proofs are omitted, they may be found in the references as indicated.

2—1. *Let $\{X_\lambda, f_{\lambda\mu}, \Lambda\}$ be an inverse system of compact metric spaces, where Λ is countable. Then X_∞ is a metric space.*