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AUTOMORPHISMS OF POSTLIMINAL C^*-ALGEBRAS

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Let $\alpha(\mathfrak{A})$ denote the group of automorphisms of a C^*-algebra $\mathfrak{A}$. The object of this paper is to give an intrinsic algebraic characterization of those elements α of $\alpha(\mathfrak{A})$ which are induced by a unitary operator in the weak closure of $\mathfrak{A}$ in every faithful representation, and it is attained for the class of C^*-algebras known as GCR, or more recently postliminal. The relevant condition is that α should map closed two-sided ideals of $\mathfrak{A}$ into themselves, and the main theorem (Theorem 2) may be thought of as an analogue for C^*-algebras of Kaplansky's theorem for von Neumann algebras, namely that an automorphism of a Type I von Neumann algebra is inner if and only if it leaves the centre elementwise fixed. The proof of Theorem 2 requires the—probably unnecessary—assumption that $\mathfrak{A}$ is separable.

By a C^*-algebra we mean a Banach algebra over the complex numbers, with a conjugate-linear anti-automorphic involution $A \to A^*$ satisfying $\| A^*A \| = \| A^* \| \cdot \| A \|$. The mappings of C^*-algebras which we consider (automorphisms, representations, etc.) will always be assumed to preserve the adjoint operation, and by a homomorphic image of a C^*-algebra $\mathfrak{A}$, we mean the image of a homomorphism from $\mathfrak{A}$ into another C^*-algebra $\mathfrak{B}$ (this is automatically a C^*-subalgebra of $\mathfrak{B}$ [2; 1.8.3]). We shall refer to Dixmier's book [2] for all standard results that we need to quote concerning C^*-algebras. By the theorem of Gelfand-Naĭmark (see, e.g. [2; 2.6.1]), a C^*-algebra has an isometric representation as an algebra of operators on a Hilbert space, and we shall usually think of a given C^*-algebra as being "concretely" represented on some Hilbert space. A *state* of a C^*-algebra $\mathfrak{A}$ is a positive linear functional of norm one. The set $\mathfrak{S}$ of states of $\mathfrak{A}$ is a convex subset of the (Banach) dual space of $\mathfrak{A}$. If $\mathfrak{A}$ has an identity element then $\mathfrak{S}$ is w^*-compact, but in any case $\mathfrak{S}$ contains an abundance of extreme points, which are called *pure states*. The set of pure states of $\mathfrak{A}$ will be denoted by $\mathfrak{P}$.

Given a state ρ of $\mathfrak{A}$, there is a representation ϕ_ρ of $\mathfrak{A}$ on a Hilbert space H_ρ, and a unit vector x_ρ in H_ρ such that $\{\phi_\rho(A)x_\rho : A \in \mathfrak{A}\}$ is dense in H_ρ (i.e. the representation ϕ_ρ is cyclic) and

$$\rho(A) = \langle \phi_\rho(A)x_\rho, x_\rho \rangle$$

for each $A \in \mathfrak{A}$. ϕ_ρ is irreducible if and only if ρ is pure. Given a state ρ of $\mathfrak{A}$, and a representation ϕ of $\mathfrak{A}$ on H, we say that ρ is a *vector state* (in the representation ϕ) if $\rho(A) = \langle \phi(A)x, x \rangle$ for some