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ON ABELIAN PSEUDO LATTICE ORDERED GROUPS

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Throughout this paper po-group will mean partially ordered abelian group. A subgroup H of a po-group G is an o-ideal if H is a convex, directed subgroup of G. A subgroup M of G is a value of $0 \neq g \in G$ if M is an o-ideal of G that is maximal without g. Let $\mathscr{M}(g) = \{M \subseteqq G \mid M$ is a value of $g\}$ and $\mathscr{M}^*(g) = \bigcap \mathscr{M}(g)$. Two positive elements $a, b \in G$ are pseudo disjoint (p-disjoint) if $a \in \mathscr{M}^*(b)$ and $b \in \mathscr{M}^*(a)$, and G is a pseudo-lattice ordered group (pl-group) if each $g \in G$ can be written $g = a - b$ where a and b are p-disjoint.

The main result of § 2 shows that every pl-group G is a Riesz group. That is, G is semiclosed ($ng \geqq 0$ implies $g \geqq 0$ for all $g \in G$ and all positive integers n), and G satisfies the Riesz interpolation property; if, whenever $x_1, \cdots, x_m, y_1, \cdots, y_n$ are elements of G and $x_i \leqq y_j$ for $1 \leqq i \leqq m$, $1 \leqq j \leqq n$, then there is an element $z \in G$ such that $x_i \leqq z \leqq y_j$.

In § 3, we determine which Riesz groups are also pl-groups. In the final section it is shown that each pair of p-disjoint elements a, b determines an o-ideal $H(a, b)$ with the property that if $a - b = x - y$ where x and y are also p-disjoint, then $H(a, b) = H(x, y)$ and $a - x = b - y \in H(a, b)$.

The concept of a pl-group has been introduced by Conrad [1]. For each $g \in G$, $\mathscr{M}^*(g)$ exists by definition, and in particular, $\mathscr{M}^*(0) = G$. In § 2 we list a number of properties of pl-groups that will be used. We adopt the notation $a \| b$ for $a \ngeqq b$ and $b \ngeqq a$. If S is a subset of a po-group G and $a \in G$, the notation $a > S$ means $a > s$ for all $s \in S$. If H is an o-ideal of a po-group G, a natural order is defined in G/H by setting $X \in G/H$ positive if X contains a positive element of G. All quotient structures will be ordered in this manner. Finally, $G^+ = \{x \in G \mid x \geqq 0\}$.

2. Some properties of pl-groups.

We first list a number of properties of pl-groups. The proofs of these may be found in [1]. If G is a pl-group, then

(1) G is semiclosed.

(2) G is directed.

(3) The intersection of o-ideals of G is an o-ideal.

(4) If $g \in G$ and $M \in \mathscr{M}(g)$ and M' is the intersection of all o-ideals of G that properly contain M, then $g \in M'$, M'/M is o-isomorphic to a naturally ordered subgroup of the real numbers and, if $M < X \in G/M \backslash M'/M$, then $X > M'/M$.