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TOTALLY GEODESIC HYPERSURFACES OF KAEHLER MANIFOLDS

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It is known that a C^∞ orientable totally umbilical hypersurface P with nonzero mean curvature of a Kaehler manifold M is a normal contact manifold. Moreover, if $M = C_n$ with the flat Kaehler metric, P can be realized as a normal contact metric manifold of positive constant curvature. It is the main purpose of this paper to obtain corresponding results for cosymplectic manifolds.

The direct product of two normal almost contact manifolds can be endowed with a complex structure. For cosymplectic manifolds more is obtained. Indeed, the direct product of two cosymplectic manifolds can be given a Kaehlerian structure. This is particularly true of orientable totally geodesic hypersurfaces of a Kaehler manifold.

Our notion of a cosymplectic manifold differs from the one given by P. Libermann in [3] and was given by D. Blair [1].

THEOREM 1. *A necessary and sufficient condition that a C^∞ orientable hypersurface P of a Kaehler manifold M be cosymplectic with almost contact form η is that its second fundamental form H be proportional to $\eta \otimes \eta$, that is*

$$H = h\eta \otimes \eta ,$$

where $h = H(\xi, \xi)$, the vector field ξ being the contravariant form of η with respect to the almost contact metric.

COROLLARY 1. *A C^∞ orientable totally geodesic hypersurface of a Kaehler manifold is a cosymplectic manifold.*

A corresponding result was obtained by Y. Tashiro [6] for totally umbilical hypersurfaces.

For complete simply connected cosymplectic manifolds an application of the de Rham decomposition theorem (see [2]) yields

COROLLARY 2. *A C^∞ complete simply connected orientable totally geodesic hypersurface of a Kaehler manifold is a product with one factor Kaehlerian.*

THEOREM 2. *A cosymplectic hypersurface of C_n with the flat Kaehler metric is locally flat.*