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QUALITATIVE BEHAVIOR OF SOLUTIONS OF A THIRD ORDER NONLINEAR DIFFERENTIAL EQUATION

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This paper investigates the behavior of nonoscillatory solutions and the existence of oscillatory solutions of the differential equation

$$y''' + p(t)y' + q(t)y^r = 0$$

where $p(t)$ and $q(t)$ are continuous and real valued on a half axis $[a, \infty)$ and r is the quotient of odd positive integers. The two cases $p(t), q(t) \leqq 0$ and $p(t), q(t) \geqq 0$ are discussed.

One theorem improves an oscillation criterion of Waltman [16]. Other results supplement those obtained by Lazar [10].

1. In this paper, real valued solutions of

$$y''' + p(t)y' + q(t)y^r = 0 \tag{1.1}$$

are investigated where $p(t)$ and $q(t)$ are continuous real valued functions defined on some interval $[a, \infty)$ with $a > 0$. Furthermore $q(t)$ is not eventually (i.e., for sufficiently large t) identically zero. r is assumed to be the quotient of odd integers. This insures that solutions with real initial conditions are real and also that the negative of a solution of (1.1) is also a solution of (1.1).

Motivation for the study of this equation comes from two directions. The equation

$$y''' + p(t)y' + q(t)y = 0$$

has been studied extensively. Some recent papers are those of Gregus [3], Hanan [5], Lazer [10], and Svec [15]. On the other hand, the equation

$$y^{(n)} + q(t)y^r = 0, \qquad n \geqq 2$$

has been investigated by Licko and Svec [11] for $r \neq 1$, by Kiguradze [9] for $r < 1$, and by Mikusinski [12] for $r = 1$. Equation (1.1) has been studied recently by Waltman [16].

A solution of (1.1) is said to be continuable if it exists on $[a_1, \infty)$ for some $a_1 \geqq a$. A nontrivial solution of (1.1) is called oscillatory if it is continuable and has zeros for arbitrarily large t. A nontrivial solution of (1.1) is called nonoscillatory if it is continuable and not oscillatory.

Two cases, $p(t) \leqq 0, q(t) \leqq 0$ and $p(t) \geqq 0, q(t) \geqq 0$ are discussed