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FINITE DIMENSIONAL TORSION FREE RINGS

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In the class of rings with zero singular left ideal, several characterizations of rings with finite left Goldie dimension are given. They include: the direct limit of torsion free modules is torsion free; the direct limit of torsion free injective modules is injective; each absolutely pure torsion free module is injective; each module has a unique (up to isomorphism) torsion free covering module. The latter result gives a converse, in a special case, to a theorem of Mark Teply.

Throughout, R will denote an associative ring with identity and module, without further qualification, will mean unitary left R-module. For a module E, we use $S \subseteq' E$ to denote that S is a large submodule of E[4, p. 60]; $Z(E)$ will denote the singular submodule of E, which consists of those elements in E whose annihilators are large left ideals in R.

DEFINITION 1. A module E is torsion free if $Z(E) = (0)$ and if $Z(R) = (0)$ we say R is a torsion free ring.

A submodule S of a module E is closed in E if $S \subseteq' T \subseteq E$ implies $T = S$. The following facts are easily verified.

LEMMA 1. *Let S be a submodule of a module E.*
(a) *If $Z(E/S) = (0)$, S is closed in E.*
(b) *If $Z(E) = (0)$, S is closed in E if and only if $Z(E/S) = (0)$.*

Proof. See Lemma 2.3 in [8].

DEFINITION 2. A module E has finite (Goldie) dimension if it contains no infinite direct sum of nonzero submodules. If the module R has finite dimension we call R a finite dimensional ring and write dim R is finite.

1. Torsion Free Rings.

Over an integral domain the direct limit of torsion free modules is torsion free. In this section we show that, in the class of torsion free rings, this property characterizes the finite dimensional rings. We also give two noetherian-like characterizations of such rings.

We record a theorem of F. Sandomierski [7] for easy reference.

THEOREM S. *Let $Z(R) = (0)$, and Q the maximal left quotient*