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ABSOLUTELY TORSION-FREE RINGS

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Call a ring A (left) *absolutely torsion-free* (ATF) if for every finite kernel functor σ (i.e., a topologizing filter of nonzero left ideals), $\sigma(A) = 0$. Since a commutative ring is ATF iff it is an integral domain, ATF rings may be viewed as generalizations of domains. Now an ATF ring is a prime ring, but there are even primitive rings that are not ATF. However if A is either finite as a module over its center, or finite dimensional and nonsingular as a left A-module, then A is ATF iff it is prime—in which case A is right ATF as well. The class of ATF rings is closed under the formation of polynomial rings, overrings in the maximal quotient ring, and Morita equivalence, but not under subrings. If A is ATF with maximal left quotient ring Q, then Q is simple, self-injective and von Neumann regular. Furthermore Q is artinian iff A is (left) finite dimensional. An interesting class of ATF rings are the hereditary noetherian prime rings (HNP). Techniques used in deriving properties of ATF rings show that every ring between an HNP ring A and its maximal quotient ring is itself a ring of quotients of A with respect to some idempotent kernel functor, and thus is HNP itself.

The ideas of kernel functors, filters of left ideals, torsion theories, etc. are by now well known, and we assume the reader is familiar with them. The terminology and notation are that of Goldman [5]. In particular, for any ring A, $K(A)$ (respectively $I(A)$) denotes the set of kernel functors (respectively idempotent kernel functors) on the category of left A-modules. Furthermore, τ_A denotes the unique largest kernel functor with respect to which A is torsion-free; τ_A is idempotent; a left ideal $\mathfrak{A}$ is τ_A-open if and only if $\mathfrak{A}$ is a rational left ideal; and $Q_{\tau_A}(A)$ is the Utumi maximal left ring of quotients of A (see [10]).

NOTE. In this paper all rings have units, ring homomorphisms are unital, and the term "module" means left module over the ring being considered. Also since we are using kernel functors defined on the category of left modules, our definition is actually that of left ATF rings.

1. Absolutely torsion-free rings.

The following proposition motivates our forthcoming definition.

PROPOSITION 1.1 *Let R be a commutative ring. Then R is an integral domain $\Leftrightarrow$ for every $\sigma \in K(R)$, $\sigma \neq \infty \Rightarrow \sigma(R) = 0$.*