PACIFIC JOURNAL OF MATHEMATICS
Vol. 46, No. 2, 1973

DUAL SPACES OF CERTAIN VECTOR SEQUENCE SPACES

RONALD C. ROSIER

This article is an investigation of certain spaces of sequences with values in a locally convex space analogous to the generalized sequence spaces introduced by Pietsch in his monograph *Verallgemeinerte Volkommene Folgenräume* (Akademie-Verlag, Berlin, 1962). Pietsch combines a perfect sequence space Λ and a locally convex space E to obtain the space $\Lambda(E)$ of all E valued sequences $x = (x_n)$ such that the scalar sequence $(\langle a, x_n \rangle)$ is in Λ for every $a \in E'$. Define $\Lambda\{E\}$ to be the space of all E valued sequences $x = (x_n)$ such that the scalar sequence $(p(x_n))$ is in Λ for every continuous semi-norm p on E. The spaces $\Lambda(E)$ and $\Lambda\{E\}$ are topologized using the topology of E and a certain collection $\mathscr{M}$ of bounded subsets of Λ^x, the $\alpha -$ dual of Λ.

The criteria for bounded sets, compact sets, and completeness are similar for both spaces. The significant difference lies in the duality theory. The dual of $\Lambda(E)_{\mathscr{M}}$ is difficult to represent, but the dual of $\Lambda\{E\}_{\mathscr{M}}$ is shown to be easily representable for general Λ and E. For many special cases of Λ and E the dual of $\Lambda\{E\}_{\mathscr{M}}$ is of the form $\Lambda^x\{E'\}$ where Λ^x is the $\alpha -$ dual of Λ and E' is the strong dual of E.

We begin by recalling basic definitions and elementary facts about sequence spaces and establishing some notation. After defining the space $[\Lambda\{E\}_{\mathscr{M}}]$ and deriving some elementary properties, we proceed to a description of its dual space. We show that the notion of a "fundamentally Λ-bounded" space E provides sufficient conditions for the duality relationship $\Lambda\{E\}' = \Lambda^x\{E\}$. We next show that there are large classes of Λ and E satisfying these conditions and we conclude by applying our results to the case $\Lambda = l^p$ obtain, for example, that the strong dual of $l^p\{E\}$ is $l^q\{E'\}$ for E a normed, Frechet, or (DF)-space, $1 \leqq p < \infty$, $p^{-1} + q^{-1} = 1$.

I would like to thank Professor G. M. Köthe for his encouragement during the preparation of this work.

2. Definitions and notations. A sequence space Λ is a vector space of real or complex sequences with the usual coordinatewise operations. To each sequence space Λ there corresponds another sequence space Λ^x, called the $\alpha -$ dual of of Λ, consisting of all $\alpha = (\alpha_n)$, such that the scalar products $\langle \alpha, \beta \rangle = \sum \alpha_n \beta_n$ converge absolutely, that is $\sum |\alpha_n \beta_n| < \infty$, for all β in Λ. Letting Λ^{xx} denote the $\alpha -$ dual of