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A TWO-POINT BOUNDARY PROBLEM FOR NONHOMOGENEOUS SECOND ORDER DIFFERENTIAL EQUATIONS

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This paper is concerned with second order nonhomogeneous differential equations, together with boundary conditions specified at two points. The existence of eigenvalues is established and the oscillatory behavior of the associated eigenfunctions is studied. The results of this paper are obtained by considering the nonhomogeneous problem without regard for existence of solutions of the associated homogeneous boundary problem.

Consider the linear differential equation

$$(r(x, \lambda)y')' + q(x, \lambda)y = f(x, \lambda) , \tag{1}$$

and the associated homogeneous equation

$$(r(x, \lambda)u')' + q(x, \lambda)u = 0 , \tag{2}$$

where $r(x, \lambda)$, $q(x, \lambda)$, and $f(x, \lambda)$ are real-valued functions on X: $a \leqq x \leqq b$, L: $\lambda_{\#} - \delta < \lambda < \lambda_{\#} + \delta$, $0 < \delta \leqq \infty$, $-\infty < a < b < \infty$. We shall consider (1) together with two-point boundary conditions of the form

$$\begin{aligned} &\text{(a)}\quad \alpha(\lambda)y(a, \lambda) - \beta(\lambda)(ry')(a, \lambda) = 0 , \\ &\text{(b)}\quad \gamma(\lambda)y(b, \lambda) - \delta(\lambda)(ry')(b, \lambda) = 0 . \end{aligned} \tag{3}$$

It is well known that for those values of λ for which the associated homogeneous boundary problem (2, 3) has no solution, the nonhomogeneous problem (1.3) yields a unique solution. Further, for those values of λ for which (2, 3) has a solution, the problem (1, 3) either has no solution or an infinite number of solutions.

In either case the homogeneous problem must be solved or shown to have only the trivial solution. This paper establishes the existence of characteristic values for (1, 3) independent of the corresponding reduced problem. The methods used will be analogous to those of W. M. Whyburn [6, 7, 8], and G. J. Etgen [2, 3].

The following hypotheses on the coefficients involved in the boundary problem will be assumed throughout:

(H_1) For each $x \in X$, each of $r(x, \lambda)$, $q(x, \lambda)$, and $f(x, \lambda)$ is continuous on L.

(H_2) For each $\lambda \in L$, each of $r(x, \lambda)$, $q(x, \lambda)$, and $f(x, \lambda)$ is measurable on X.