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ON THE FROBENIUS RECIPROCITY THEOREM FOR SQUARE-INTEGRABLE REPRESENTATIONS

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In this paper, a global version of the Frobenius reciprocity theorem is established for irreducible square-integrable representations of locally compact unimodular groups. As in the classical compact case, it asserts that certain intertwining spaces are canonically and isometrically isomorphic. The proof is elementary, and the appropriate isomorphism is exhibited explicitely. The essential point is that square-integrability implies the continuity of functions in certain subspaces of L^2 spaces on which the group acts and leads to a characterization of the subspaces in terms of reproducing kernels.

The preliminary results on reproducing kernels are contained in Theorems 1 and 2 in § 2. Our main result on reciprocity, Theorem 3 in § 3, does not require direct integral decomposition theory as in [2] and [4] and is formally similar to the version of the reciprocity theorem proved by C. C. Moore in [5]; however, we only consider unitary representations, and do not need to formulate the result in terms of summable induced representations on L^1-spaces.

After this paper was initially submitted, we learned that A. Wawrzyńczyk [6] had already proved a result, similar but not identical to our Theorem 3. His proof is based on a general duality theorem for automorphic forms due to K. and L. Maurin [3], and he does not prove results corresponding to our Theorems 1 and 2.

Let G be a locally compact unimodular group and S a continuous irreducible square-integrable unitary representation of G on a complex Hilbert space $\mathscr{H}$. We recall that this implies

$$x \longrightarrow (S(x)\varphi \mid \psi),\ x \in G$$

is square-integrable on G for all φ and ψ in $\mathscr{H}$ and the existence of a positive constant d (the formal degree) such that

$$\int_G (S(x)\varphi \mid \alpha)\,\overline{(S(x)\psi \mid \beta)}dx = d^{-1}(\varphi \mid \psi)\overline{(\alpha \mid \beta)} \tag{1.1}$$

for all φ, α, ψ, β in $\mathscr{H}$.

Let K be a compact subgroup of G and λ a continuous irreducible unitary representation of K on a complex-Hilbert space $\mathscr{K}$. Let $T = T(\cdot, \lambda)$ be the continuous unitary representation of G induced by λ. By definition, $T(y)(y \in G)$ is right translation by y on the space $L^2(G, \lambda)$ of all square-integrable maps $f: G \longrightarrow \mathscr{K}$ such that