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NONZERO SOLUTIONS TO BOUNDARY VALUE PROBLEMS FOR NONLINEAR SYSTEMS

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We are mainly concerned here with solutions of

$$x' = A(t, x)x + F(t, x) , \tag{*}$$

which satisfy the following conditions

$$x \in B , \quad x(t) \not\equiv 0 . \tag{1.1}$$

Here $A(t, u)$ is a real $n \times n$ matrix defined and continuous on $J \times R^n$, where J is a subinterval of $R = (-\infty, \infty)$. The real n-vector $F(t, u)$ is also defined and continuous on $J \times R^n$. In (1.1) B is a Banach space of continuous functions on J.

Two theorems are given concerning the solution to the above problem in the case of a finite interval J. The first theorem (Th. 3.1) deals with the homogeneous system

$$x' = A(t, x)x , \tag{1.2}$$

and the second (Th. 4.1) is concerned with the system (*) with a *small* perturbation $F(t, u)$. The third result of this paper (Th. 5.1) extends to rather heavily nonlinear systems a result of Medvedev [12] dealing with the existence of nontrivial, nonnegative solutions of (*) on $[0, \infty)$. Medvedev considered the case of a perturbed linear system. The method employed here is a comparison technique. In other words, for each function $f \in M$ (a certain compact, convex set of functions in B) we assume the existence of solutions in M of the linear system

$$x' = A(t, u(t))x + F(t, u(t)) , \tag{1.u}$$

and then we apply a fixed point theorem for multi-valued mappings, due to Eilenberg and Montgomery [4], in order to ensure the existence of solutions in M of the equation (*). As far as the author knows, the first application of the above fixed point theorem was given by Schmitt [14], who considered the case $B = \{x \in C([0, \omega]); x(0) = x(\omega)\}$, and a linear second order scalar equation with deviating arguments.

For results related to the contents of this paper, the reader is referred to Lasota, Opial [11], Opial [13], Avramescu [1], [2], Corduneanu [3] and Kartsatos [7], [8].

2. Preliminaries.

Let J be a subinterval of $R = (-\infty, \infty)$.