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CONNECTEDNESS IM KLEINEN AND LOCAL CONNECTEDNESS IN 2^X AND $C(X)$

JACK T. GOODYKOONTZ, JR.

Let X be a compact connected metric space and $2^X(C(X))$ denote the hyperspace of closed subsets (subcontinua) of X. In this paper the hyperspaces are investigated with respect to point-wise connectivity properties. Let $M \in C(X)$. Then 2^X is locally connected (connected im kleinen) at M if and only if for each open set U containing M there is a connected open set V such that $M \subset V \subset U$ (there is a component of U which contains M in its interior). This theorem is used to prove the following main result. Let $A \in 2^X$. Then 2^X is locally connected (connected im kleinen) at A if and only if 2^X is locally connected (connected im kleinen) at each component of A. Several related results about $C(X)$ are also obtained.

A continuum X will be a compact connected metric space. $2^X(C(X))$ denotes the hyperspace of closed subsets (subcontinua) of X, each with the finite (Vietoris) topology, and since X is a continuum, each of 2^X and $C(X)$ is also a continuum (see [5]).

One of the earliest results about hyperspaces of continua, due to Wojdyslawski [7], was that each of 2^X and $C(X)$ is locally connected if and only if X is locally connected. As a point-wise property, local connectedness is stronger than connectedness im kleinen, which in turn is stronger than aposyndesis. The author [1] has shown that if X is any continuum, then each of 2^X and $C(X)$ is aposyndetic. It is the purpose of this paper to investigate the internal structure of 2^X and $C(X)$ with respect to these properties. In particular, we determine necessary and sufficient conditions (in terms of the neighborhood structure in X) that 2^X be locally connected at a point and that 2^X be connected im kleinen at a point. We also determine that $C(X)$ has, in general, stronger point-wise connectivity properties that either 2^X or X.

For notational purposes, small letters will denote elements of X, capital letters will denote subsets of X and elements of 2^X, and script letters will denote subsets of 2^X. If $A \subset X$, then A^* (int A) (bd A) will denote the closure (interior) (boundary) of A in X.

Let $x \in X$. Then X is locally connected (l.c.) at x if for each open set U containing x there is a connected open set V such that $x \in V \subset U$. X is connected im kleinen (c.i.k.) at x if for each open set U containing x there is a component of U which contains x in its interior. X is aposyndetic at x if for each $y \in X - x$ there is a