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ON CHARACTERIZING CERTAIN CLASSES OF FIRST COUNTABLE SPACES BY OPEN MAPPINGS

KENNETH ABERNETHY

This paper has three main results. These are characterizations of Nagata spaces, γ-spaces, and semi-metric spaces, respectively, as images of metrizable spaces under certain kinds of continuous open mappings.

1. Introduction. A basic area of research in general topology is the study of how various classes of spaces are related through mappings (see [3] and [5]). More specifically, many important classes of spaces have been characterized as the image of a metrizable space under an open continuous mapping of some sort. For example, Heath [10] has characterized developable spaces in this way and Hanai and Ponomarev independently have given an elegant characterization of first countable spaces (see Theorem 2.1). In recent years considerable attention has been given to the problem of characterizing generalized metrizable spaces in this way. We mention some of these results in § 2. In this paper we characterize Nagata, semi-metric, and γ-spaces as the image of a metrizable space under certain types of open continuous mappings. Definitions and some known results are given in § 2, Nagata spaces are characterized with Theorem 3.3, γ-spaces with Theorem 4.3, and semi-metric spaces with Theorem 5.3. Throughout the paper the set of natural numbers will be denoted by N.

2. Definitions and background results. The spaces which interest us in this paper can be described in terms of sequences of open covers. It should be pointed out that many of the definitions which follow are not the original definitions, but are actually characterizations which were proved, after the particular concept had been introduced, in efforts to unify the various concepts. Consequently, the definitions we give, in terms of a *COC*-function, display some degree of this unification.

Let (X, T) be a topological space and let g be a function from $N \times X$ into T. Then g is called a *COC-function for X* ($COC=$ countably many open covers) if it satisfies these two conditions: (1) $x \in \bigcap_{n=1}^{\infty} g(n, x)$ for all $x \in X$; (2) $g(n+1, x) \subseteq g(n, x)$ for all $n \in N$ and $x \in X$. Note that if g is a *COC*-function for X, we obtain countably many open covers of X, $\langle G_n \rangle$, by taking $G_n = \{g(n, x) \colon x \in X\}$ for each n.

Now let X be a space with *COC*-function g, and consider the