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ENTIRE SOLUTIONS OF LINEAR ELLIPTIC EQUATIONS WITH LAPLACIAN PRINCIPAL PART

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Consider the equations in R^n, $n \geqq 2$,

$$(*) \qquad \Delta\varphi = f + b \cdot \nabla\varphi$$

$$(\mathrm{I}) \qquad \Delta\varphi = b \cdot \nabla\varphi$$

where f and b are locally Hölder continuous, and as $|x| \to \infty$, $f(x) = O(|x|^{-\tau})$, $b(x) = O(|x|^{-\sigma})$, σ, $\tau > 1$. It is shown that if $0 \leqq \rho < \sigma - 1$, there is a one-to-one correspondence between entire C^2 solutions of $(*)$ whose gradients grow no faster than $O(|x|^\rho)$, and harmonic polynomials with gradients of the same growth. For (I) therefore solutions whose gradients grow no faster than $O(|x|^\rho)$ form a finite dimensional vector space. These results for (I) give analogues to the concept of "generating pairs" in pseudo-analytic function theory.

1. Introduction. In the case $n = 2$, (I) takes the form

$$(1.1) \qquad \varphi_{xx} + \varphi_{yy} = b_1\varphi_x + b_2\varphi_y .$$

If we make the identifications $w = \varphi_x - i\varphi_y$, $A = (b_1 + ib_2)/4$, $B = (b_1 - ib_2)/4$, then w satisfies the complex equation

$$(1.2) \qquad \frac{\partial w}{\partial \bar{z}} = Aw + B\bar{w}$$

where

$$\frac{\partial}{\partial \bar{z}} = \frac{1}{2}\left(\frac{\partial}{\partial x} + i\frac{\partial}{\partial y}\right) .$$

Thus (1.1) can be studied alternatively by considering the complex equation (1.2), the solutions of which are known as "pseudo-analytic functions," and for which an extensive theory has been developed (see for example the treatments of Bers and Vekua in **[1]** and **[8]**). In particular it is known that entire and bounded solutions of (1.2) form a two-dimensional real vector space, and a basis for this vector space is called a "generating pair." In dimensions higher than two the reduction of (I) to a first order complex equation is no longer