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LUMPY SUBSETS IN LEFT-AMENABLE LOCALLY COMPACT SEMIGROUPS

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This note adapts to locally compact, topologically left-amenable semigroups S the characterization given by Mitchell in discrete semigroups of those subsets T of S substantial enough that at least one topologically left-invariant mean in S is concentrated in T.

Introduction. Mitchell (1965) showed that for a discrete left-amenable semigroup S the following two properties of a subset T of S are equivalent:

(1) There is at least one left-invariant mean α on the space $m(S)$ of bounded functions on S which is concentrated on T; that is, $\alpha(\xi_T) = 1$.

(2) T is left-thick; that is, for each finite subset σ of S there is an s in S such that $\sigma s = \{ts \mid t \in \sigma\}$ is contained in T.

My paper (Day, 1969, Th. 7.2) gave a version of this for locally compact *groups*, which Wong (preceding paper) points out has a misprint; $K \backslash Es$ should have been $K \cap Es$ in condition (2) of that theorem.

In a locally compact *group* G there are (at least) three conditions equivalent to left amenability of G; that is, existence of a left-invariant mean α on the bounded measurable functions on G. The pertinent three are called in Day (1968) conditions (lsau), (lsam), and (lsaπ) and will be defined in the next section. In locally compact semigroups each implies the next, but they are not known to be equivalent except in groups. In the preceding paper Wong chose a condition (W) equivalent to (lsau) and generalized Mitchell's theorem to semigroups satisfying (W). This note picks the simpler middle property (lsam) and proves a Mitchelloid theorem for such semigroups. For the weakest of the properties (lsaπ) some implications are given but no equivalent condition is yet known.

NOTATION. C_0 is the space of continuous functions vanishing at infinity on S, a locally compact semigroup, and M is the space of regular Borel measures on S. [It is well known that M is like the conjugate space of C_0; see Hewitt and Ross (1963).] $P \subset M$ is the set of probability measures and $P_c \subset P$ is the set of probability measures with compact support, so P_c is dense in P. $P^{**} \subset M^{**}$ is the set of means on M^*; as usual, the canonical image $Q(P_c)$ of P_c into M^{**} is w^*-dense in P^{**}.

C_0 and M^* are abstract M-spaces and M and M^{**} are abstract