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A REPRESENTATION THEOREM FOR CONVOLUTION TRANSFORM WITH DETERMINING FUNCTION IN L^p.

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Let $G(t)$ be a kernel in Class II. Necessary conditions in order that a function $f(x)$ be the convolution transform of $\phi(t) \in L^p(-\infty, \infty)$ were obtained by the second author. Also it was conjectured that the conditions are in fact sufficient. The conjecture is indeed true and we prove it here.

Following the notation of [3] (see [3] §2) we have

THEOREM. *Necessary and sufficient conditions in order that $f(x)$ possess the representation*

$$f(x) = \int_{-\infty}^{\infty} G(x-t)\phi(t)dt, \qquad \gamma < x < \infty$$

where $\phi(t) \in L^p(-\infty, \infty)$ $(1 < p < \infty)$ are that $f(x) \in C^\infty(\gamma, \infty)$ and that

$$\sup_{\gamma < x < \infty} \sum_{n=0}^{\infty} \frac{1}{a_{n+1}} |f_n(x - \lambda_n)|^p \equiv H < \infty. \tag{1}$$

Furthermore,

$$\int_{-\infty}^{\infty} |\phi(t)|^p dt = H. \tag{2}$$

Necessity follows from [3], Theorem 2 for $M(u) = |u|^p$. The equality (2) is established in the proofs of Theorems 2, 3 in [3]. For the sufficiency we shall need the following lemmas,

LEMMA 1. *For every $\tau > 0$,*

$$\sum_{n=0}^{\infty} \frac{1}{a_{n+1}} H_{n+1}(-\tau + \lambda_{n+1}) = 1.$$

Proof. By [3] (3.10),

$$\sup_{-\infty < \theta < \infty} \sum_{n=0}^{\infty} \frac{1}{a_{n+1}} G_n(\theta - \lambda_n) \leqq 1.$$