PACIFIC JOURNAL OF MATHEMATICS
Vol 111, No. 2, 1984

GENERALIZED COMPLETE MAPPINGS, NEOFIELDS, SEQUENCEABLE GROUPS AND BLOCK DESIGNS. I

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The necessary and sufficient condition that the latin square formed by the Cayley multiplication table of a group has an orthogonal mate is that the group has a complete mapping. Here, we define two generalizations of the concept of a complete mapping and show how these generalizations are related to sequenceable groups and R-sequenceable groups respectively and that together they permit a complete characterization of left neofields. In the second part of the paper, we shall show that these generalizations also yield new constructions of block designs of Mendelsohn type.

Introduction. In [**11**] H. B. Mann introduced the concept of a *complete mapping* of a finite group $(G, \cdot)$ and showed that, when a group has such a complete mapping, the latin square formed by its Cayley multiplication table has a transversal and, hence, an orthogonal mate. See also §1.4 of [**1**]. Later, L. J. Paige [**13**] showed that complete mappings can also be used in the construction of neofields. This fact has been used extensively in [**6**].

More recently, it has been shown in [**10**] and [**2**] that a necessary condition for a finite group to be R-sequenceable is that it possess a complete mapping. Similarly, we prove below that a necessary condition for a finite group to be *sequenceable* is that it possess a *near complete mapping*. In general, both sequenceable and R-sequenceable groups permit the construction of neofields and the neofields so constructed are of special type.

In this first part of our paper, we define two generalizations of the concept of a complete mapping to be called a (K, λ) *complete mapping* and a (K, λ) *near complete mapping*, respectively, and we show that all of the above-mentioned concepts can be described in terms of these generalizations. We are also able to give a complete characterization for all left neofields. In the second part (to appear shortly), we show that these generalizations also yield new constructions of block designs of Mendelsohn type whose automorphism group contains a specified subgroup and we describe in more detail how (K, λ) generalized complete mappings and near complete mappings may be constructed.