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FINITE SUBGROUPS OF SU_2, DYNKIN DIAGRAMS AND AFFINE COXETER ELEMENTS

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Dedicated to the memory of my friend Ernst Straus

Using, among other things, some properties of affine Coxeter elements, for which we also present normal forms, we offer an explanation of the McKay correspondence, which associates to each finite subgroup of SU_2 an affine Dynkin diagram.

J. McKay [**M**] has observed that for each finite (Kleinian) subgroup G of SU_2 the columns of the character table of G, one column for each conjugacy class, form a complete set of eigenvectors for the corresponding affine Cartan matrix (of type A_n, D_n or E_n), the one that arises in connection with the resolution of the singularity of $\mathbf{C}^2/G$ at the origin (see 1(9) below). As he has observed, this follows at once from: if ρ is the two-dimensional representation by which G is defined, $\{\rho_i\}$ is the set of (complex) irreducible representations of G, and $\sum n_{ij}\rho_j$ denotes the decomposition of $\rho \otimes \rho_i$, then $C \equiv [c_{ij}] \equiv [2\delta_{ij} - n_{ij}]$ is the relevant Cartan matrix. Partial explanations have been given by several authors (see [**G**], [**H**], [**K**], [$\mathbf{S}_1$, Appendix III]). Here we shall give our own explanation of this and some related facts, including two normal forms for affine Coxeter elements which enter into our considerations. Section 1 details mainly with McKay's correspondence, Section 2 mainly with affine Coxeter elements. As general references for Kleinian groups, Kleinian singularities and root systems, we cite [**C**, Chapters 7, 11], [$\mathbf{S}_1$, Section 6], [**B**], and the survey article [$\mathbf{S}_2$].

1. In this section G is a finite group, ρ is a faithful (complex) representation of G of finite dimension d, $\{\rho_i\}$ is the set of all irreducible representations of G with ρ_0 the trivial one, $\sum n_{ij}\rho_j$ denotes the decomposition of $\rho \otimes \rho_i$, and C is the matrix $[d\delta_{ij} - n_{ij}]$.

(1) The column $[\chi_j(x)$ (x in G fixed, $j = 1, 2, \ldots)]$ of the character table of G is an eigenvector of C with $d - \chi(x) = \chi(1) - \chi(x)$ as the corresponding eigenvalue. In particular $[d_1, d_2, \ldots]$ ($d_i = \dim \rho_i$) is an eigenvector corresponding to the eigenvalue 0.