Differential and Integral Equations, Volume 4, Number 2, March 1991, pp. 225–239.

FEEDBACK CONTROLLABILITY OF THE FREE BOUNDARY OF THE ONE PHASE STEFAN PROBLEM

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1. Introduction. Let Ω be a bounded open subset of $\mathbb{R}^n$ with a sufficiently smooth boundary Γ and let $\sigma \in C^2(\overline{\Omega_1})$ be a given function in $\Omega_1 \subset \Omega$ such that

$$\Omega_t = \{x \in \Omega : \sigma(x) < t\}$$

is increasing in $t \in [0,T]$, $\sigma(x) = 0\ \forall\, x \in \Omega_0$ and $\|\nabla\sigma(x)\| \neq 0\ \forall\, x \in \Omega\backslash\Omega_0$.

We set

$$Q = \{(x,t) \in \Omega\times]0,T[; \sigma(x) < t < T\},$$

$$\Sigma = \Gamma\times]0,T[; \quad \Sigma_0 = \{(x,t) \in \Omega\times]0,T[; t = \sigma(x)\},$$

$\Omega_T = \Omega_1$ and

$$Q_\varepsilon = \{(x,t) \in \Omega\times]\varepsilon,T[;\ \sigma(x) < t < T\}$$

for any $\varepsilon \in]0,T[$.

Consider the following controllability problem:

(P_0) *Given σ find $u \in L^2(Q)$ such that*

$$\begin{cases} \dfrac{\partial y}{\partial t} - \Delta y = u & \text{in } Q \\ \langle \nabla y, \nabla\sigma\rangle = \rho, \ \ y = 0 & \text{in } \Sigma_0 \\ y = 0 & \text{in } \Sigma \\ y(0,x) = y_0(x) & \text{in } \Omega_0 \\ y \leq 0 & \text{in } Q. \end{cases} \tag{1}$$

Here ρ is a positive constant and $y_0 \in L^2(\Omega_0)$ is a given function.

System (1) is the one phase Stefan problem describing a solidification process having Σ_0 as the interface which separates the solid and the liquid phase. The process is controlled by a distributed source u in Q. (P_0) is an inverse Stefan problem

Received September 21, 1990.
AMS Subject Classifications: 49A15, 49A35.