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RESEARCH ANNOUNCEMENTS

A QUILLEN STRATIFICATION THEOREM FOR MODULES

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Let G be a finite group and k a fixed algebraically closed field of characteristic $p > 0$. If p is odd, let H_G be the subring of $H^*(G, k)$ consisting of elements of even degree; take $H_G = H^*(G, k)$ if $p = 2$. H_G is a finitely generated commutative k-algebra, and we let V_G denote its associated affine variety Max H_G. If M is any finitely generated kG-module, the *cohomology variety* $V_G(M)$ of M may be defined as the support in V_G of the H_G-module $H^*(G, M)$ if G is a p-group, and in general as the largest support of $H^*(G, L \otimes M)$ where L is any kG-module. A module L with each irreducible kG-module as a direct summand will do [3].

D. Quillen [9, 10] proved a number of beautiful results relating V_G to the varieties V_E associated with the elementary abelian p-subgroups E of G, culminating in his stratification theorem. This theorem gives a piecewise description of V_G in terms of the subgroups E and their normalizers in G. Some of Quillen's results have been extended to the variety $V_G(M)$ associated with a kG-module M [1, 4, 5, 6, 7, 8], and the work of Alperin and Evens [2] and Avrunin [3] showed that there was at least a surjection $\amalg_E V_E(M) \longrightarrow V_G(M)$. However, the stratification theorem for $V_G(M)$ remained elusive, since one still needed to know that a point in $V_G(M)$ in the image of a given V_E was in fact in the image of $V_E(M)$.

We announce here a proof of the stratification theorem for $V_G(M)$, as well as a proof of a conjecture of J. Carlson regarding $V_E(M)$ for E an elementary abelian p-subgroup. We are also able to generalize several of Quillen's other results to the module case.

For $H < G$, let $t_{G,H}\colon V_H \longrightarrow V_G$ be the transfer map induced by restriction on the cohomology rings. For an elementary abelian p-subgroup E, let $V_E^+ = V_E \backslash \bigcup_{F<E} t_{E,F}V_F$ and let $V_E^+(M) = V_E^+ \cap V_E(M)$. Then put $V_{G,E}^+(M) = t_{G,E}V_E^+ \cap V_G(M)$. We have the following stratification theorem.

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