

FUNDAMENTAL REGIONS FOR THE SIMPLE GROUP OF ORDER 60 IN S_4

R. R. STOLL

Introduction. By a finite collineation group of order g , a point will be transformed into g points at the most. A set of points which are permuted among themselves by the operators of the group are said to form a conjugate set of points. A fundamental region in the plane for the group is defined as a system of points which contains one and only one point from every conjugate set. Therefore, for a collineation group of order g , there are g fundamental regions.

For example, consider the group of order four generated by the transformations $x = y'$, $y = -x'$. These transformations consist of the rotations of a point through one, two, three, and four right angles about the origin in the xy -plane. Any two perpendicular lines through the origin divide the plane into four parts which are the fundamental regions for the group on the plane. Points on the two lines themselves may be assigned to either of the two adjacent regions, provided that no point and its transform be put in the same fundamental region. This example demonstrates that the solution of such a problem is not unique.

The definition of fundamental regions is readily extended to groups in more than two variables, and in the case of three variables the problem has received some attention. J. W. Young* obtained a solution for cyclic groups, W. I. Miller† for the G_{168} , and H. F. Price‡ for the G_{24} and the G_{60} when written with real coefficients in every transformation. This paper is devoted to the determination of fundamental regions for the G_{60} when written with complex coefficients in its transformations. The method used is similar to that used by Miller for the G_{168} .

Determination of fundamental regions. Considered as a collineation group, the generating substitutions for the G_{60} are taken to be§

* J. W. Young, *Fundamental regions for cyclic groups of linear fractional transformations on two complex variables*, this Bulletin, vol. 17 (1910–1911), pp. 340–344.

† W. I. Miller, *Fundamental regions for the simple group of order 168 in S_4* , American Journal of Mathematics, vol. 56 (1934), pp. 316–318.

‡ H. F. Price, *Fundamental regions for certain finite groups in S_4* , American Journal of Mathematics, vol. 40 (1919), pp. 108–113.

§ H. F. Blichfeldt, *Finite Collineation Groups*, University of Chicago Press, 1917, chap. 5, p. 114.