A remark on a construction of Grundhöfer

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Abstract

We consider a large class of "natural" generalizations of Grundhöfer's synthetic definition of incidence in Figueroa planes. We determine which of these generalized definitions produce projective planes and which do not. We show that those few which do produce projective planes produce only Pappian planes or Figueroa planes.

A class of non-Desarguesian, proper, finite projective planes of orders q^3 for prime powers $q \not\equiv 1 \pmod 3$ and $q > 2$ was defined by Figueroa [4] in 1982. This construction was generalized to all prime powers $q > 2$ by Hering and Schaeffer [6] later in the same year. We [2] gave a group-coset description of these finite Figueroa planes in 1983. The construction was extended to include infinite planes in 1984 by Dempwolff [3]. These constructions were all algebraic in the sense that they made essential use of collineation groups and coordinates.

In 1986 Grundhöfer [5] gave a beautiful synthetic construction which included all these Figueroa planes. It is very tempting to try to generalize this synthetic construction. However, we prove that certain kinds of generalizations of Grundhöfer's construction are impossible.

Grundhöfer's construction begins with a Pappian projective plane Π which has an order three planar automorphism α. Points and lines of Π are of three types, with respect to α, according to the structure of their orbits under $\langle\alpha\rangle$. A point P is of type-I if $P^\alpha = P$, it is of type-II if P, P^α and P^{α^2} are collinear and distinct, and it is of type-III if P, P^α and P^{α^2} are noncollinear. The types of lines are defined

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