On partitions, surjections, and Stirling numbers

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1 Introduction

The connection between $P(m,n)$, the number of partitions of a set containing m elements as a disjoint union of n non-empty subsets; $S(m,n)$, the number of surjections of a set of m elements onto a set of n-elements; and $St(m,n)$, the Stirling number of the second kind, given by[1]

$$St(m,n) = \frac{1}{n!}\sum_{r=0}^{n}(-1)^{n-r}\binom{n}{r}r^m, \tag{1.1}$$

has long been known (see, eg, [B, C, L, T]). Indeed, their mutual relation is given by

$$\frac{1}{n!}S(m,n) = P(m,n) = St(m,n), \tag{1.2}$$

the first equality being very elementary and the second somewhat less immediate.

Our primary object in this paper is to provide an explicit formula for $St(m,n)$, and hence, by (1.2), for $P(m,n)$ and $S(m,n)$, in the case that $m > n$.

[1]Other definitions are given in the litterature; for example, $St(m,n)$ is characterized by the equation

$$x^m = \sum_{n=0}^{m} n!St(m,n)\binom{x}{n}.$$

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