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Injective spectra with respect to the K-homologies

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§0. Introduction

Given a CW-spectrum E we call a CW-spectrum W *E_*-injective* if any map $f: X \to Y$ induces an epimorphism $f^*: [Y, W] \to [X, W]$ whenever $f_*: E_*X \to E_*Y$ is a monomorphism [12, Definition 1 i)]. The well known ring spectra $E = S$, HZ/p, MO, MU, MS_p, KU, KO and KT satisfy some of nice properties as stated in [1] or [2]. For example, E_*E is flat as an E_*-module, and the product map $v_{E,F}: E_*E \underset{E_*}{\otimes} \pi_*F \to E_*F$ is an isomorphism for any E-module spectrum F. Then E_*X may be regarded as a comodule over the coalgebra E_*E. For such a nice ring spectrum E we gave the following characterization in [17].

THEOREM 1. *Let E be a ring spectrum satisfying the above two properties. For a CW-spectrum W the following conditions are equivalent:*
i) *W is an E_*-injective spectrum,*
ii) *W is an E_*-local spectrum such that E_*W is injective as an E_*E-comodule, and*
iii) *the canonical morphism $\kappa_E: [X, W] \longrightarrow \mathrm{Hom}_{E_*E}(E_*X, E_*W)$ is an isomorphism for any CW-spectrum X.*

In this note we study $K_*^{\mathscr{C}}$-injective spectra for $K^{\mathscr{C}} = KU \vee KO \vee KT$, $KU \vee KO$, $KU \vee KT$, $KO \vee KT$, KU, KO and KT where KU, KO and KT denote the complex, the real and the self-conjugate K-spectrum respectively. In particular, we give a $K_*^{\mathscr{C}}$-version of Theorem 1 as our main result (see Theorem 2 below). For our purpose we use the Bousfield's abelian categories CRT and $ACRT$ [9, 2.1 and 5.5] whose objects $M = \{M^C, M^R, M^T\}$ are modelled on the united K-homologies $K_*^{CRT}X = \{KU_*X, KO_*X, KT_*X\}$ for any CW-spectra X, although our category $ACRT$ is somewhat different from the Bousfield's one.

In §1 we first recall the abelian category CRT and then state several homological properties of CRT established in [9, §2 and §3] for later use. In §2 we introduce the abelian categories $\mathscr{C} = CR$, CT, RT, C, R and T whose objects $M = \{M^H\}$ are obtained by restricting their namesakes in CRT, of which CR and C have already been done in [9, 4.1 and 4.7]. In §3 we give