HIROSHIMA MATH. J.
5 (1975), 487–497

Remarks on Extendible Vector Bundles over Lens Spaces and Real Projective Spaces

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(Received May 19, 1975)

§1. Introduction

Let K be a CW-complex and L be its subcomplex. A real (resp. complex) vector bundle ζ over L is said to be *extendible* to K if ζ is equivalent to the restriction of a real (resp. complex) vector bundle over K.

R. L. E. Schwarzenberger ([2], [6]) studied the extendibility of vector bundles over CP^n (resp. RP^n) to CP^m (resp. RP^m), $m>n$, where CP^n (resp. RP^n) is the complex (resp. real) projective n-space.

The purpose of this paper is to establish some results concerning the extendible real vector bundles over the standard lens space $L^n(p)=S^{2n+1}/Z_p$ and the real projective space by the somewhat different methods. Our main results are as follows.

THEOREM 1.1. *Let p be an odd prime and ζ be a real t-plane bundle over $L^n(p)$. Assume that there is a positive integer l satisfying the following properties*:

(i) *ζ is stably equivalent to a sum of $[t/2]+l$ non-trivial real 2-plane bundles.*

(ii) $p^{[n/(p-1)]} > [t/2]+l$.

Then $n<2[t/2]+2l$ and ζ is not extendible to $L^m(p)$ for each $m\geqq 2[t/2]+2l$.

THEOREM 1.2. *Let p be any integer >1. The tangent bundle $\tau(L^n(p))$ of $L^n(p)$ is extendible to $L^{n+1}(p)$ if and only if $n=0$, 1 or 3.*

We also obtain the results (Theorems 6.2 and 6.6) for RP^n corresponding to the above theorems.

In §2, we recall the structure of K-ring of $L^n(p)$ according to T. Kambe [3], which is useful in §3 for the proof of Theorem 1.1. In §4, we have sufficient conditions for the existence of the extension of a real vector bundle over $L^n(p)$ (Theorems 4.2 and 4.3) and give an example of a real t-plane bundle over $L^n(p)$ which is extendible to $L^{m-1}(p)$ but not to $L^m(p)$ $(m=2[t/2]+2l)$. The proof of Theorem 1.2 is carried out in §5. Also, we give an example of an extendible vector bundle over $L^n(p)$ which shows that the condition (ii) of Theorem 1.1 cannot