HIROSHIMA MATH. J.
6 (1976), 343-351

Some Z_q-Equivariant Immersions

Dedicated to Professor Kiiti Morita on his 60th birthday

Teiichi KOBAYASHI

(Received January 14, 1976)

§ 1. Introduction

Let q be an integer and Z_q be the cyclic group of order q. A C^∞-differentiable immersion f of a Z_q-manifold in another Z_q-manifold is called a *Z_q-equivariant immersion* (or simply a *Z_q-immersion*) if f is a Z_q-equivariant map. The purpose of this note is to study the conditions for the existence of some Z_q-equivariant immersions.

Let m and k be non-negative integers, and R^{m+2k} be Euclidean $(m+2k)$-space. Let $R^{m,2k}$ be the Z_q-manifold (R^{m+2k}, Z_q) with the action

$$\mu\colon Z_q \times R^{m+2k} \longrightarrow R^{m+2k}$$

defined by

$$\mu(T, (t_1,\dots, t_m, z_{m+1},\dots, z_{m+k})) = (t_1,\dots,t_m, Tz_{m+1},\dots, Tz_{m+k}),$$

where $T(=e^{2\pi\sqrt{-1}/q})$ is the generator of Z_q, $t_1,\dots, t_m$ are real numbers $(\in R)$, and $z_{m+1},\dots, z_{m+k}$ are complex numbers $(\in C = R^2)$.

Let S^{2n+1} be the unit $(2n+1)$-sphere in complex $(n+1)$-space C^{n+1}. Let (S^{2n+1}, Z_q) be the Z_q-manifold defined by the action

$$\nu\colon Z_q \times S^{2n+1} \longrightarrow S^{2n+1};\ \nu(T, (z_0,\dots, z_n)) = (Tz_0,\dots, Tz_n),$$

where $z_0,\dots, z_n$ are complex numbers with $\sum_{j=0}^{n}|z_j|^2 = 1$. The action ν is free and differentiable of class C^∞. The orbit manifold S^{2n+1}/Z_q is the standard lens space $L^n(q) \bmod q$.

A. Jankowski obtained in [7] some non-existence theorems for Z_2-immersions. In this note we consider Z_q-immersions $(S^{n+1}, Z_q) \to R^{m,2k}$, and study the bounds of m for fixed k and n.

As is easily seen, there is a Z_q-immersion of (S^{2n+1}, Z_q) in $R^{m,0}$ if and only if there is an immersion of $L^n(q)$ in R^m.

If $k > n$, (S^{2n+1}, Z_q) is Z_q-immersible in $R^{m,2k}$ for any m, clearly. In case $k \leqq n$, we have the following results.

THEOREM 1. *Let q be an integer >1. Then (S^{2n+1}, Z_q) is not Z_q-immer-*