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Matlis duality and the width of a module

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Introduction

The aim of this paper is to study the Matlis duality and some related topics concerning artinian modules over a commutative noetherian ring.

In § 1, we give some results on the Matlis duality. First, we generalize the Matlis duality which is known for noetherian local rings to the case of noetherian semi-local rings. Secondly, we examine the problem on the self-duality. As is well-known, a finite dimensional vector space over a field k (resp. a finite abelian group) is isomorphic to its Matlis dual, i. e. its k-dual (resp. its character group). We determine the class of noetherian rings for which the self-duality holds with respect to the Matlis duality. It turns out that, in case of domains, it characterizes the class of rings of the above-mentioned type.

§ 2 is preparatory. We prove some properties of attached primes, the notion of which has been recently introduced by I. G. Macdonald and R. Y. Sharp.

In § 3, we define *coregular sequences*, the *width* of a module and the *cograde* of a module. These are dual notions to those of regular sequences, the depth of a module and the grade of a module respectively. The first two notions have been already introduced (in different terminologies) by E. Matlis (cf. [5]). We investigate, by using the results of § 2, some properties of these notions. Especially, we characterize the cograde (resp. the width) by the vanishing of Tor modules, and the relationships between the cograde and the grade (resp. the width and the depth) with respect to the Matlis duality are established. Finally, we calculate the width of certain local cohomology modules.

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§1. Matlis duality

1. Throughout this section, we denote by A a commutative noetherian ring with unit. Let E_A be the module $\oplus E_A(A/\mathfrak{m})$, where $\mathfrak{m}$ runs over the set of maximal ideals of A. Then E_A is an injective cogenerator for A; namely, (a) E_A is injective and (b) any A-module can be embedded in a product of E_A (cf. Sharpe, Vámos [7] Chap. 2). Let $\mathscr{C}$ denote the category of A-modules of finite length.

PROPOSITION 1.1. *Suppose that D is a contravariant, left-exact, A-linear*