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*A Note on Finite Groups which Act Freely on Closed Surfaces II**

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§1. Introduction

This note is a continuation of the previous note [2].

Let T_m or U_m be the orientable or non-orientable closed surface of genus m. In [2], we studied finite groups which act freely on the Klein bottle U_2 and the torus T_1, and on T_m preserving the orientation. In this note, we study what kind of finite groups can act freely on U_m, and on T_m reversing the orientation. Here we say that a finite group G acts on T_m reversing the orientation if some element of G reverses the orientation of T_m.

Let F_n be the free group generated by $x_1, \dots, x_n$, and set $s_n = \prod_{i=1}^n x_i^2 \in F_n$. We say that an element w of F_n is even if w is a product of even times of generators, i.e., a form $\prod_{j=1}^{2k} x_{i_j}$, and is odd if it is not even; and also a subgroup K of F_n is even if any element of K is even, and is odd if it is not even. Also we denote by ${}^{\#}G$ the order of a finite group G. Then we have the following propositions.

PROPOSITION 1.1 (cf. [2, Prop. 3.2]). (i) *A finite group G acts freely on T_m reversing the orientation if and only if there exists an even normal subgroup K of F_n such that*

$$(1.2) \qquad G \cong F_n/K, \quad K \ni s_n, \quad 2(1-m) = (2-n)({}^{\#}G).$$

For this case, the orbit surface T_m/G is homeomorphic to U_n.

(ii) *A finite group G acts freely on U_m if and only if there exists an odd normal subgroup K of F_n such that*

$$(1.3) \qquad G \cong F_n/K, \quad K \ni s_n, \quad 2-m = (2-n)({}^{\#}G).$$

For this case, U_m/G is homeomorphic to U_n.

PROPOSITION 1.4 (cf. [2, Prop. 3.3]). (i) *Let G be a finite 2-group and assume that the minimum number of generators of G is n. Then G acts freely on T_m reversing the orientation, where $m = 1 + (n-1)({}^{\#}G)$.*

(ii) *Let G be a finite group and assume that the number of generators of G is less than $n+1$. Then G acts freely on U_m, where $m = 2 + (2n + l - 2)({}^{\#}G)$*

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