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Realizing Some Cyclic BP_*-modules and Applications to Stable Homotopy of Spheres

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Introduction

Let $BP_*(\ )$ be the Brown-Peterson homology theory localized at a prime $p \geqq 5$. Its coefficient ring BP_* is the polynomial ring $Z_{(p)}[v_1, v_2, \ldots]$ over the integers localized at p on Hazewinkel's generators v_i of degree $2(p^i-1)$ ([2], [3], [4], [6]).

In the previous paper [14; Th. D, DII, D', D'II], we constructed the spectra realizing cyclic BP_*-modules $BP_*/(p, v_1^j, v_2^{sp})$ at $p \geqq 5$ in the following three cases: $1 \leqq j \leqq p$, $s \geqq 1$, $(j, s) \neq (p, 1)$; $p+1 \leqq j \leqq 2p-2$, $p|s$; $p+1 \leqq j \leqq 2p$, $2p|s$. In this paper, we shall prove the following realizability theorems.

THEOREM 4.3. *For $p \geqq 5$ and $s \geqq 2$, there exist spectra L_s such that* $BP_*(L_s) = BP_*/(p^2, v_1^p, v_2^{sp^2})$.

THEOREM 4.4. *For $p \geqq 5$, $s \geqq 2$ and j with $p+1 \leqq j \leqq 2p$, there exist spectra $Y_{s,j}$ such that* $BP_*(Y_{s,j}) = BP_*/(p, v_1^j, v_2^{sp^2})$.

Each L_s is an 8-cell complex and we define the element $\beta_{sp^2/(p,2)}$ in $\pi_*(S)$, the stable homotopy group of spheres, by the attaching map of the 5th cell at the 4th cell in L_s, and similarly we define $\beta_{sp^2/(j)} \in \pi_*(S)$ from $Y_{s,j}$ (for the details, see Definitions 5.1–5.2). Then using methods developed by H. R. Miller, D. C. Ravenel, W. S. Wilson and others ([7], [8], [9]), we see that the elements $\beta_{sp^2/(p,2)}$ and $\beta_{sp^2/(j)}$ of the same name in $H^2BP_* = \mathrm{Ext}^{2,*}_{BP_*BP}(BP_*, BP_*)$ [8] survive nontrivially to E_∞ term in the Adams-Novikov spectral sequence and support the homotopy elements of the above.

THEOREM 5.3. *For $p \geqq 5$, $s \geqq 2$, the elements $\beta_{sp^2/(p,2)}$ in $\pi_{(sp^3+sp^2-p)q-2}(S)$ $(q=2(p-1))$ are nontrivial of order p^2 and indecomposable. Hence the group $\pi_{(sp^3+sp^2-p)q-2}(S)$ contains a summand isomorphic to Z/p^2Z.*

THEOREM 5.4. *For $p \geqq 5$, $s \geqq 2$, $p+1 \leqq j \leqq 2p$, the elements $\beta_{sp^2/(j)}$ in $\pi_{(sp^3+sp^2-j)q-2}(S)$ $(q=2(p-1))$ are indecomposable and generate cyclic summands of order p.*

The known elements in $\pi_*(S)$ of order p^2 are the elements in $\mathrm{Im}\, J$ [1] and the