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*Partite-Claw-Decomposition of a Complete Multi-Partite Graph**)

Shinsei TAZAWA, Kazuhiko USHIO and Sumiyasu YAMAMOTO

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1. Introduction

A multi-partite graph, denoted by $G_m(n_1, n_2,..., n_m)$, is a graph whose point set can be partitioned into m subsets $V_1, V_2,..., V_m$ with $n_1, n_2,..., n_m$ points each, such that every line joins different subsets. If it contains every line joining different subsets, then it is called a complete m-partite graph and is denoted by $K_m(n_1, n_2,..., n_m)$. A complete graph K_m with m points may be regarded as a particular type of complete m-partite graph where $n_1=n_2=\cdots=n_m=1$. A complete bipartite graph $K_2(1, c)$ or a tree with $c+1$ points and radius one is called a claw or a star of degree c.

A claw of degree c being a subgraph of a multi-partite graph will be called a *partite-claw* (PC) of degree c if no pair of points lies in the same set of points of the multi-partite graph.

A graph is called claw-decomposable if it can be decomposed into a union of line-disjoint claws of the same degree. The problem of claw-decomposability of a complete graph K_m has been raised and solved completely by Yamamoto, Ikeda, Shige-eda, Ushio and Hamada [4]. The claw-decomposition of a complete graph provides us an optimal balanced file organization scheme of order two, called **HUBFS**$_2$, for binary-valued records in a sense such that it has the least redundancy among all possible balanced binary-valued file organization schemes of order two having the same parameters, provided the distribution of records has the property of invariance with respect to the permutation of attributes [3].

An analogous theorem which states a necessary and sufficient condition for the claw-decomposability of a complete m-partite graph $K_m(n, n,..., n)$ where $n_1=n_2=\cdots=n_m=n$ has been obtained by Ushio, Tazawa and Yamamoto [2].

In this paper, a theorem which states a necessary and sufficient condition for the decomposability of such a complete m-partite graph $K_m(n, n,..., n)$ into a union of line-disjoint partite-claws of degree c, which will be called a PC-decomposition theorem of the m-partite graph, will be given. An algorithm for the de-

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