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Existence of Various Boundary Limits of Beppo Levi Functions of Higher Order

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1. Introduction

Let U be the unit open ball with center at the origin in the n-dimensional Euclidean space R^n. It is well known that a non-negative function u harmonic in U has a non-tangential limit at almost every boundary point of U. Diederich [4] proved that such a function u has an mc (mean continuous) limit at almost every point of ∂U; we say that u has an mc limit ℓ at $\xi \in \partial U$ if

$$\lim_{r \downarrow 0} \frac{1}{r^n} \int_{B(\xi, r) \cap U} |u(x) - \ell| \, dx = 0,$$

$B(\xi, r)$ being the open ball with center at ξ and radius r. By the mean value property of harmonic functions, we can show easily that u has a non-tangential limit at every point of ∂U at which u has an mc limit.

Now let f be a function defined on U whose (partial) derivatives of the first order exist a.e. in U and satisfy

$$\text{(a)} \qquad \int_U |\operatorname{grad} f|^2 (1 - |x|)^\alpha dx < \infty, \qquad 0 \leqq \alpha < 1.$$

This condition only does not necessarily ensure the existence of non-tangential limits of f (see Proposition 2 in Sec. 4). In case $n=2$, assuming that f is continuous in U, Carleson [2; Theorem 3 in Sec. V] proved the existence of radial limits of f. Wallin [19; Theorem 1] generalized Carleson's theorem to higher dimensional case with f defined on the upper half space R^n_+ and satisfying the condition analogous to (a):

$$\text{(b)} \qquad \int \cdots \int_G |\operatorname{grad} f|^2 x_n^\alpha dx_1 \cdots dx_n < \infty, \qquad 0 \leqq \alpha < 1,$$

for any bounded open set $G \subset R^n_+$. He also proved that if in addition f is harmonic in R^n_+, then the non-tangential limit of f exists at $\xi \in \partial R^n_+$ except for a set whose Riesz capacity of order $2-\alpha$ is zero ([19; Theorem 3]).

In this paper we are concerned with Beppo Levi functions f of order m defined on R^n_+ which satisfy