HIROSHIMA MATH. J.
10 (1980), 141–152

Extremal solutions of general nonlinear differential equations

Kurt KREITH and Takaŝi KUSANO

(Received August 21, 1979)

Recently, one of the present authors [4] has studied various forms of maximal and minimal asymptotic behavior of positive solutions of the nonlinear differential equations

$$y^{(n)} + f(t, y) = 0, \quad y^{(n)} - f(t, y) = 0.$$

This paper extends the results of [4] to much more general differential equations of the form

$$(1^+) \qquad L_n y + f(t, y) = 0$$

$$(1^-) \qquad L_n y - f(t, y) = 0$$

where $n \geq 2$ and

$$(2) \qquad L_n = \frac{1}{p_n(t)} \frac{d}{dt} \frac{1}{p_{n-1}(t)} \frac{d}{dt} \cdots \frac{d}{dt} \frac{1}{p_1(t)} \frac{d}{dt} \frac{\cdot}{p_0(t)} .$$

It also establishes criteria for the absence of various forms of asymptotic behavior among the eventually positive solutions of (1^+) and (1^-) and, in some cases, the complete absence of eventually positive solutions.

We always assume that:

$$(3) \qquad \begin{aligned} &\text{(a)} \quad p_i \in C([a, \infty), (0, \infty)), \quad 0 \leq i \leq n; \\ &\text{(b)} \quad f \in C([a, \infty) \times (0, \infty), (0, \infty)). \end{aligned}$$

We introduce the notation:

$$(4) \qquad L_0 y(t) = \frac{y(t)}{p_0(t)}, \quad L_i y(t) = \frac{1}{p_i(t)} \frac{d}{dt} L_{i-1} y(t), \quad 1 \leq i \leq n.$$

The domain $\mathscr{D}(L_n)$ of L_n is defined to be the set of all functions y: $[T_y, \infty) \to R$ such that $L_i y(t)$, $0 \leq i \leq n$, are continuous on $[T_y, \infty)$. By a positive solution of (1^+) $[(1^-)]$ we mean a function $y \in \mathscr{D}(L_n)$ which satisfies (1^+) $[(1^-)]$ and is positive for all sufficiently large t.