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On the group of fibre homotopy equivalences

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Introduction

Let (E, p, B, F) denote a Hurewicz fibration with projection $p: E \to B$ and fibre F. Then the set of all free fibre homotopy classes of free fibre homotopy equivalences of E to itself forms a group under the multiplication defined by the composition of maps. This group is called the group of fibre homotopy equivalences of a Hurewicz fibration (E, p, B, F), and we denote it by $\mathscr{L}(E)$.

The group $\mathscr{L}(E)$ has been studied by several authors, e.g., [5], [6], [15], [16], [19], [21], [24] and [33]. We notice that for any covering space, this is the group of all covering transformations.

The purpose of this paper is to study the group $\mathscr{L}(E)$ of a Hurewicz fibration (E, p, S^n, F) over the n-sphere S^n $(n \geqq 1)$, where the fibre F is assumed to be a locally compact CW-complex. Let aut F denote the H-space of all free homotopy equivalences of F to itself with the identity map $1: F \to F$ as the base point. Then we may consider a Hurewicz fibration

$$(1) \qquad (E_k, p, S^n, F) \text{ with characteristic map } k \in \pi_{n-1}(\operatorname{aut} F),$$

because any fibration (E, p, S^n, F) is freely fibre homotopy equivalent to such a fibration by a classification theorem due to Stasheff [25, Th. 1.5–1.6] (for details, see §§ 1–2).

Now let $\mathscr{F}(F) = \pi_0(\operatorname{aut} F)$ be the group of all free homotopy classes of free homotopy equivalences of F to itself, and consider the action of $\mathscr{F}(F)$ on the homotopy group $\pi_i(\operatorname{aut} F)$ by the conjugation denoted by $\cdot$ (see § 1). Then, by using Gottlieb's theorem ([5, Th. 1]), we can prove the following basic theorem of this paper in Theorem 2.2 and Corollary 2.5:

THEOREM I. *For the group $\mathscr{L}(E_k)$ of fibre homotopy equivalences of a fibration* (1), *there holds the exact sequence*

$$\pi_1(\operatorname{aut} F) \xrightarrow{\partial_k} \pi_n(\operatorname{aut} F) \xrightarrow{G} \mathscr{L}(E_k) \xrightarrow{J_0} \mathscr{F}_k(F) \longrightarrow 1,$$

where ∂_k is given by the Samelson product: $\partial_k(x) = \langle k, x \rangle$, $\mathscr{F}_k(F) = \{\alpha \in \mathscr{F}(F) | \alpha \cdot k = k\}$, and J_0 is the homomorphism obtained by the restriction to the fibre F.

Especially, for the trivial fibration $(F \times S^n, p, S^n, F)$ which is the one of (1) *with $k=0$, this sequence is the split exact sequence*