HIROSHIMA MATH. J.
14 (1984), 203–210

A class of hyperbolic focal point problems

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(Received August 15, 1983)

1. Introduction

The hyperbolic equation

$$u_{tt} - u_{ss} + p(s, t)u = 0 \qquad (p>0) \tag{1.1}$$

has the physical interpretation of prescribing the u-displacement of a vibrating string subject to a linear restoring force. This fact suggests the possibility of formulating a hyperbolic boundary value problem by specifying boundary conditions for the string at times $t=0$ and $t=T$. In analogy with the ordinary differential equation

$$\frac{d^2u}{dt^2} + p(t)u = 0 \qquad (p>0) \tag{1.2}$$

(describing the u-displacement of a single particle), one might expect that such boundary values will also give rise to eigenvalues for hyperbolic equations of the form

$$u_{tt} - u_{ss} + \lambda p(s, t)u = 0. \tag{1.3}$$

While there have been a number of hyperbolic generalizations of the Sturm comparison theorem (see for example [2], [3], [5], [6]) which can be useful in this regard, these extensions of classical ODE results require careful attention to boundary conditions in the space variable s as well as in t. The present paper attempts to avoid the complications associated with spatial boundary conditions by considering focal point problems for (1.1) in characteristic triangles of the form

$$R(s, t) = \{(\sigma, \tau): s-(t-\tau)\leq\sigma\leq s+(t-\tau), \quad 0\leq\tau\leq t\}.$$

For example, we shall study (1.3) in $R(0, T)$ with various boundary conditions assigned at $t=0$ and at $t=T$.

It is assumed throughout that $p(s, t)$ is continuous in $R(0, T)$ and that all solutions are C^2 functions which satisfy the underlying equation in the classical sense.