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On decaying entire solutions of second order sublinear elliptic equations

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1. Introduction

In this paper we consider the semilinear ellitpic equation

$$\Delta u + a(x)u^{\sigma} = 0 \quad \text{in} \quad R^n, \tag{1}$$

where $n \geqq 3$, $x=(x_1,\dots, x_n)$, $\Delta=\sum_{i=1}^{n} \partial^2/\partial x_i^2$, $0<\sigma<1$, and $a(x)$ is a positive locally Hölder continuous function (with exponent $\alpha \in (0, 1)$) in R^n.

We are interested in the existence of positive entire solutions of equation (1). By an entire solution of (1) we mean a function $u \in C_{loc}^{2+\alpha}(R^n)$ which satisfies (1) at every point of R^n. The problem of existence of such solutions has been studied by several authors including [1–5]. Most of them have dealt with bounded positive entire solutions which are bounded away from zero. However, equation (1) may also have positive entire solutions which approach zero as $|x| \to \infty$.

The main objective of this paper is to prove the existence of positive entire solutions of (1) decaying to zero at infinity. Our procedure is to construct solutions of (1) which are squeezed between supersolutions and subsolutions tending to zero as $|x| \to \infty$. The latter are obtained as spherically symmetric solutions of elliptic equations with $a(x)u^{\sigma}$ in (1) replaced by radial majorants and minorants. For this purpose we need a global existence theory of a certain singular boundary value problem for nonlinear ordinary differential equations. We also attempt to extend the main result for (1) to semilinear elliptic systems of the form

$$\begin{cases} \Delta u + a(x)u^{\sigma}v^{\tau} = 0 \\ \Delta v + b(x)u^{\lambda}v^{\mu} = 0, \end{cases} \tag{2}$$

where σ, τ, λ and μ are nonnegative constants and $a(x)$ and $b(x)$ are positive locally Hölder continuous functions in R^n.

2. Main results

We employ the notation:

$$a^*(r) = \max_{|x|=r} a(x), \qquad a_*(r) = \min_{|x|=r} a(x). \tag{3}$$