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Positive solutions of linear and quasilinear elliptic equations in unbounded domains

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1. Introduction

Let Ω be an exterior domain in $\boldsymbol{R}^N$, $N \geqq 2$, with smooth boundary $\Gamma = \partial\Omega$ and let $\mathfrak{D}$ and B denote, respectively, an elliptic differential operator and a boundary operator defined by

$$\mathfrak{D} = \textstyle\sum_{i,j=1}^{N} a_{ij}(x)\partial^2/\partial x_i \partial x_j + \sum_{i=1}^{N} b_i(x)\partial/\partial x_i, \quad x \in \Omega, \tag{1.1}$$

and

$$B = \alpha(x)\partial/\partial\beta + (1-\alpha(x))\cdot, \quad x \in \Gamma, \tag{1.2}$$

where $\partial/\partial\beta$ is the directional derivative in the direction of a vector β prescribed on Γ. We are concerned with the following linear and quasilinear boundary value problems:

$$-\mathfrak{D}u + c(x)u = \lambda m(x)u \quad \text{in } \Omega, \quad Bu = 0 \quad \text{on } \Gamma, \tag{A}$$

$$-\mathfrak{D}u + c(x)u = \lambda m(x)u^{\gamma} \quad \text{in } \Omega, \quad Bu = 0 \quad \text{on } \Gamma, \tag{B}$$

where $c(x)$ and $m(x)$ are given functions, λ is a real parameter and γ is a nonzero constant with $\gamma \neq 1$. We allow Γ to be empty, in which case Ω is the entire space $\boldsymbol{R}^N$ and the boundary condition in (A) or (B) is void.

The objective of this paper is twofold. First, we study the existence and asymptotic behavior of positive functions h which satisfy the differential inequality

$$-\mathfrak{D}h + c(x)h \geqq \lambda m(x)h \quad \text{in } \Omega \tag{1.3}$$

and have minimal order of growth at infinity. Such an h is called a minimal λ-superharmonic function, and the totality of λ-superharmonic functions is denoted by $SH(\lambda)$. An analysis of some particular cases of (1.3) ([10]) shows that the asymptotic behavior of λ-superharmonic functions is in general very complicated. So, we restrict our attention to the situations in which (i) all h in $SH(\lambda)$ converge to zero as $|x| \to \infty$; (ii) all h in $SH(\lambda)$ are bounded both above and below by positive constants; (iii) all h in $SH(\lambda)$ tend to infinity as $|x| \to \infty$, and attempt to obtain conditions for such situations to occur. For this purpose a