HIROSHIMA MATH. J.
15 (1985), 117–125

Two-step methods with two off-step nodes

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(Received August 16, 1984)

1. Introduction

Consider the initial value problem

$$y' = f(x, y), \quad y(x_0) = y_0, \tag{1.1}$$

where $f(x, y)$ is assumed to be sufficiently smooth. Let $y(x)$ be the solution of this problem,

$$x_n = x_0 + nh \qquad (n = 1, 2, \dots; h > 0), \tag{1.2}$$

where h is a stepsize. Let y_1 be an approximation of $y(x_1)$ obtained by some appropriate method. We are concerned with the case where the approximations y_j $(j=2, 3, \dots)$ of $y(x_j)$ are computed by two-step methods. Conventional two-step methods such as linear two-step methods [1], pseudo-Runge-Kutta methods [1, 3] and so on [4] require starting values y_0 and y_1 to generate y_j $(j=2, 3, \dots)$.

In our previous paper [5], introducing a set of subsidiary off-step nodes

$$x_{n+\nu} = x_0 + (n+\nu)h \qquad (n = 0, 1, \dots; 0 < \nu < 1), \tag{1.3}$$

at the cost of supplying an additional starting value y_ν, we proposed two-step methods for computing y_{n+1} together with subsidiary approximations $y_{n+\nu}$ of $y(x_{n+\nu})$ $(n=1, 2, \dots)$. It has been shown that for $r=2, 3$ there exists such a method of order $r+3$ with r function evaluations per step.

In this paper we introduce another set of off-step nodes

$$x_{n+\mu} = x_0 + (n+\mu)h \qquad (n = 0, 1, \dots; 0 < \mu < 1, \mu \neq \nu) \tag{1.4}$$

and at the expense of providing one more starting value y_μ we propose two-step methods of the form

$$y_{n+\mu} = y_n + b_{r+1}(y_n - y_{n-1}) + h \textstyle\sum_{j=0}^{r} c_{r+1\,j} k_{jn} \qquad (r = 3, 4, 5), \tag{1.5}$$

$$y_{n+\nu} = y_n + b_{r+2}(y_n - y_{n-1}) + h \textstyle\sum_{j=0}^{r+1} c_{r+2\,j} k_{jn}, \tag{1.6}$$

$$y_{n+1} = y_n + s(y_n - y_{n-1}) + h \textstyle\sum_{j=0}^{r+2} p_j k_{jn}, \tag{1.7}$$

where