Hiroshima Math. J.
16 (1986), 499–530

Global existence of mild solutions to semilinear differential equations in Banach spaces

Toshiyuki Iwamiya

(Received September 20, 1985)

Introduction

Let X be a Banach space over the real field $\boldsymbol{R}$ with norm $|\cdot|$. Let $\{S(t); t\geqq 0\}$ be a linear contraction semigroup on X of class (C_0) and let A be the infinitesimal generator of $\{S(t); t\geqq 0\}$. Let Ω be a subset of $[a, b)\times X (a<b\leqq +\infty)$ and let B be a nonlinear continuous operator from Ω into X.

In this paper we are concerned with the existence and uniqueness of global mild solutions to the initial-value problem for a semilinear differential equation in X

$$(0.1) \qquad u'(t) = Au(t) + B(t, u(t)), \quad \tau < t < b, \quad u(\tau) = z,$$

where (τ, z) is given in Ω. Here by a mild solution is meant an X-valued continuous function u on the interval $[\tau, b)$ satisfying the following Volterra integral equation:

$$(0.2) \qquad u(t) = S(t-\tau)z + \int_\tau^t S(t-s)B(s, u(s))ds, \quad \tau \leqq t < b.$$

In general, a mild solution may not be differentiable and hence need not be an exact solution to (0.1). But this notion is known as the most natural one of the generalized notions of solutions to (0.1). For regularity results of mild solutions, see for instance Martin [10].

Semilinear equations of type (0.1) have been studied by many authors and the present paper is related to the works of Iwamiya [1], Kato [2], [3], Kenmochi and Takahashi [4], Lakshmikantham et al [6], Lovelady and Martin [7], Martin [8], [9], Pavel [11], [12], [13], Pavel and Vrabie [14], [15] and Webb [16].

In case Ω is open, various results have been obtained by the analogy with the theory of ordinary differential equations in $\boldsymbol{R}^n$. The case in which Ω is closed has been considered in relation to so-called flow invariant sets.

For the case in which $A=0$ and equation (0.1) is understood to be an ordinary differential equation in a cylindrical domain $\Omega=[a, b)\times D$ in the product space $[a, b)\times X$, Martin established fundamental results. A properly noncylindrical case was studied by Kenmochi and Takahashi [4] and their results have been recently generalized by Iwamiya [1].