HIROSHIMA MATH. J.
17 (1987), 281–307

The effect of non-local convection on reaction-diffusion equations

Shin-Ichiro EI

(Received September 17, 1986)

1. Introduction

We are concerned with the following reaction-diffusion-convection equation:

$$(1.1)\quad u_t = \left\{u_x - \int_I K(x-y)u(t,\,y)dy\cdot u\right\}_x + \varepsilon f(u),\ x\in I \equiv (-1/2,\,1/2),\ t>0$$

subject to the boundary conditions

$$(1.2)\qquad u_x - \int_I K(x-y)u(t,\,y)dy\cdot u = 0 \quad \text{at} \quad x = \pm 1/2$$

and the initial condition

$$(1.3)\qquad u(0,\,x) = u_0(x) \geqq 0,\quad x\in I.$$

This is a proto-type of spatially aggregating population models of biological individuals in a one dimensional finite habitat I, which was first proposed by Kawasaki [3] and discussed Nagai and Mimura [5] ($\varepsilon=0$) and Mimura and Ohara [4] ($\varepsilon>0$) in the whole interval $-\infty<x<\infty$. Here $u=u(t,\,x)$ represents the population density at time t and position x. From an ecological point of view, the convection velocity in the right hand side of (1.1) is specified as

$$\int_I K(x-y)\cdot u(y)dy = \int_x^{1/2} K(x-y)\cdot u(y)dy + \int_{-1/2}^{x} K(x-y)\cdot u(y)dy,$$

where $K(x)$ is an appropriate function satisfying $K(x)<0$ (resp. >0) for $x>0$ (resp. $x<0$). One knows that when

$$\int_x^{1/2} K(x-y)\cdot u(y)dy + \int_{-1/2}^{x} K(x-y)\cdot u(y)dy > 0 \quad (\text{resp.} <0).$$

the individuals move to the right (resp. left) direction. This indicates that the individuals move toward the region of higher distribution. For the growth term $\varepsilon f(u)$, we assume that ε is a sufficiently small constant, which implies that the dispersal process is very fast compared with the growth process of the species. For the ecological interpretation, see Shigesada [7]. The boundary conditions