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On locally pseudo-valuation domains

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Introduction

The purpose of this paper is to study locally pseudo-valuation domains which are quasinormal domains. In particular, we give some results on locally pseudo-valuation semigroup rings. Throughout this paper all rings are assumed to be commutative with identity.

Pseudo-valuation domains (shortly, PVD's) were introduced by J. R. Hedstrom and E. G. Houston in [9]. Also, locally pseudo-valuation domains (shortly, LPVD's) were introduced by D. E. Dobbs and M. Fontana in [4]. Examples of LPVD's are all Prüfer domains, some instances of the $D+M$ construction (cf. [5]) and certain subrings of a number field.

In the first section, we will consider the relation between the LPVD's and i-domains which were defined by I. J. Papick. In particular, we shall characterize an LPVD with the property that its integral closure is a Prüfer domain in terms of seminormality. We also note that a one dimensional Noetherian domain with finite integral closure is an LPVD if and only if it is quasinormal.

In the final section, we will give the main result. Let R be an integral domain, let S be a commutative monoid, with operation written additively, and let $R[S]$ be a monoid ring of S over R. We give a result on the problem of determining conditions under which $R[S]$ is an LPVD.

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Notation and terminology

Let R be a commutative ring with identity. We let $\mathrm{Spec}(R)$ and $\mathrm{Max}(R)$ stand for the set of all prime ideals of R and that of all maximal ideals of R respectively. An overring of R is a subring between R and its total quotient ring $Q(R)$. $\boldsymbol{Z}$, $\boldsymbol{Q}$, $\boldsymbol{Z}_0$ and $\boldsymbol{Q}_0$ denote respectively the ring of rational integers, the field of rational numbers, the set of nonnegative rational integers and the set of nonnegative rational numbers. We denote by $\bar{R}$ the integral closure of R and denote by (R, M) the quasilocal ring R with the maximal ideal M.