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Existence and qualitative theorems for nonnegative solutions of a similinear elliptic equation

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In this paper we study a qualitative feature of positive solutions for the Dirichlet problem

$$(0.1)\qquad \begin{aligned} &\Delta u(x) + f(u(x)) = 0 \qquad &&\text{in} \quad B_R \\ &u(x) = 0 &&\text{on} \quad \partial B_R, \end{aligned}$$

where $B_R = (x \in \mathbf{R}^N; |x| < R\}$, $N \geq 2$ and f is a continuous function on $[0, \infty)$ which satisfies the following conditions:

(A1) $\limsup_{s \to +0} f(s)/s \leq -m < 0$.

(A2) There exists a unique $\zeta_0 \in (0, \infty)$ such that $F(\zeta_0) = 0$, $F(\zeta) < 0$ for $\zeta \in (0, \zeta_0)$ and $f(\zeta_0) > 0$, where $F(\zeta) = \int_0^\zeta f(s)ds$,

(A3) $\alpha = \sup\{\zeta < \zeta_0; f(\zeta) = 0\}$ and $\beta = \inf\{\zeta > \zeta_0; f(\zeta) = 0\}$ exist and $0 < \alpha < \beta < \infty$.

(A4) f is Lipschitz continuous in a neighborhood of β.

We first establish an existence of positive radially symmetric solutions of (0.1) and study their shape. Hence they satisfy the following ordinally differential equation associated to (0.1)

$$(0.2)\qquad \begin{aligned} &u'' + \frac{N-1}{r} u' + f(u) = 0 \qquad \text{for} \quad 0 < r < R, \\ &u(0) = \mu, \qquad u'(0) = u(R) = 0, \end{aligned}$$

where u is now a function of $r = |x|$ alone $(x \in \mathbf{R}^N)$. Then we show the following

THEOREM 1. *Under the conditions* $(A1)$–$(A4)$ *there exists an* $R_0 > 0$ *such that for any* $R > R_0$ *the equation* (0.2) *admits a positive solution with properties*

$$\zeta_0 < u(0) < \beta \text{ and } u' < 0 \text{ on } (0, R].$$

THEOREM 2. *Let* $R = \infty$ *and define* $u(\infty)$ *by* $\lim_{r \to \infty} u(r)$. *Under the conditions* $(A1)$–$(A4)$ *for some* $\mu \in (\zeta_0, \beta)$ *there exists a nonnegative solution* u *of* (0.2). *Let*