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L^p boundedness of rough Marcinkiewicz integral on product torus

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ABSTRACT. This paper is a continuation of our study [D] [CDF] on rough Marcinkiewicz integral operator on product space. Suppose that $\Omega(x', y') \in L^q(S^{n-1} \times S^{m-1})$ $(n \geq 2, m \geq 2, q \geq 1)$ is homogeneous of degree zero satisfying the mean zero properties (1.1)–(1.3). For C^∞ functions $\tilde{f}$ on the product torus $\mathbf{T}^n \times \mathbf{T}^m$, the Marcinkiewicz integral operator on $\mathbf{T}^n \times \mathbf{T}^m$ is defined by

$$\tilde{\mu}_\Omega \tilde{f}(x, y) = \left(\int_{\mathbf{R}} \int_{\mathbf{R}} |\tilde{\Phi}_{t,s} * \tilde{f}(x, y)|^2 dt ds\right)^{1/2},$$

where $\tilde{\Phi}_{t,s}$ has the Fourier series

$$\tilde{\Phi}_{t,s}(x, y) \sim \sum_{k_1, k_2} \hat{\Phi}(2^t k_1, 2^s k_2) e^{2\pi i k_1 \cdot x} e^{2\pi i k_2 \cdot y}.$$

In this paper we show that if $q > 1$ then the operator $\tilde{\mu}_\Omega$ can be extended to a bounded operator on $L^p(\mathbf{T}^n \times \mathbf{T}^m)$ for $1 < p < \infty$.

§ 1. Introduction and results

Let $\mathbf{R}^n$ be n-dimensional Euclidean space and S^{n-1} be the unit sphere in $\mathbf{R}^n$ $(n \geq 2)$ equipped with normalized Lebesgue measure $d\sigma = d\sigma(x')$, where $x' = x/|x|$ for $x \neq 0$. In [S], Stein introducted the Marcinkiewicz integral operator μ_Ω of higher dimension as follows.

$$\mu_\Omega f(x) = \left(\int_0^\infty |F_t(x)|^2 \frac{dt}{t^3}\right)^{1/2},$$

where

$$F_t(x) = \int_{|x-y| \leq t} \frac{\Omega(x-y)}{|x-y|^{n-1}} f(y) dy,$$

$\Omega \in L^1(S^{n-1})$ is homogeneous of degree zero satisfying $\int_{S^{n-1}} \Omega(x') d\sigma(x') = 0$.

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